Designing tasks for a more inclusive school mathematics

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In this contribution to Theme A (tools and representations), we detail our approach to designing tasks to be incorporated into inclusive mathematics learning scenarios. These scenarios also involve tools created to represent mathematical knowledge in forms appropriable by students with sensory disabilities and which are developed to privilege multimodal experiences of mathematical objects, relationships and properties. We begin by introducing the theoretical influences which underpin the processes of task design and our attempts to take into consideration the complex relationships between artefacts, their mathematical affordances and the embodied practices they engender in the context of task resolution. We go on to outline the collaborative approach we adopt to simultaneously develop both tasks and tools, and how teachers, students and researchers bring different, complementary, expertise to this process. To further illustrate our approach, we consider two examples from our work with blind learners and deaf learners.

Keywords: Collaborative design, Inclusion, Blind learners, Deaf learners, Tool mediation, Embodied cognition.

Introduction

In recent years, Brazil has experienced large changes in the educational paradigm. One of these relates to the growing influence of political and social movements that defend inclusive education, and the organization of schools to attend the diverse needs of all students, without any kind of discrimination or segregation. Inclusive schools are those which see difference as a factor which enriches the educational process. They aim to support all learners in overcoming barriers to learning as they become participants in a more equitable system. The political policies related to the process of including students with special educational needs have resulted in a significant increase in their presence within mainstream schools, with statistical data from the most recent school census showing an increase of 234% between 2003 and 2010. At the same time, these policies of inclusion have been associated with taking the educational community out of the “comfort” zone and, amongst the many uncertainties, insecurities and conflict the actors in these communities are facing, questions related to curriculum demands and pedagogical actions have a central role. In particular, the increasing diversity of students within the same classroom setting raises questions related to task design: what principles and procedures might be adopted in the design of tasks for the inclusive mathematics classroom?

It is within this context that we began work on a research project aiming to (1) investigate forms of accessing and expressing mathematics which respect the diverse needs of all our students, (2) contribute to the development of teaching strategies which recognize this diversity, and (3) explore the relationships between sensory experience and mathematical knowledge[1]. The project involves the development and analysis of inclusive scenarios for mathematics learning, though a collaborative process involving researchers, teachers and students. In this article, we intend to exemplify our approach to task design and present some examples from our work in São Paulo schools.

In this approach, the process of task design is accompanied simultaneously by the development of the material and digital tools that are also incorporated in the learning scenarios. Together, tasks and tools aim to enable the interaction of different students with mathematical objects and relationships. To this end, they are designed to facilitate multiple ways of interacting with these objects and relations and to respect the diverse experiences of the students with whom we work. The approaches we use involve representing mathematical ideas through colour, sound, music, movement and texture, and hence appeal to different sensory canals, and particularly to the skin, the ears and the eyes. The multimodal nature of the mathematical representations reflects our attempts to offer stimuli appropriate to the particularities of each and every student: for those with visual impairments, the tools enable tactile and auditory stimuli, for deaf learners, tactile and visual approaches are privileged and students who can both hear and see have access to all three modes, allowing even those with specific difficulties in learning mathematics to have a variety of ways to think mathematically. Before describing in more detail the process behind the design of task and tools, we begin by delineating our understanding of the term “task” and how this term figures in our view of learning mathematics.

Learning scenarios: Tasks and activity

Our view is that tasks represent one of the elements that compose the learning scenarios we enact within. In a similar way to Laborde (2002), we see learning scenarios as consisting of specific tasks, or sequences of interrelated tasks, the mediating tools available for their resolution, along with the activities of the participating actors (which may include different combinations of students, teachers and researchers). More precisely, we distinguish between task and activity in the same ways as Dejours (1997, p.39). He argues that “a task is that which is to be achieved or that which must be done. Activity is, in the context of the task, that which is actually done by the operator to arrive as close as possible to objectives fixed by the task”. That is, tasks are proposed to the collective, and might be realized by differing individuals in different ways. Dejours (1997) was concerned with the work context and how different people might employ different techniques to attain a particular objective, depending on the tools available, of course, but also on the individuals themselves. The same is also the case in educational settings. The task proposed to a group of mathematics students might be the same, but the interpretation of the task and the activity that results will be shaped by both intrapersonal and interpersonal aspects of the particular students involved. The actions of each of the individuals who engage in a particular given task, individually or collectively, are a function, not only of the task itself, or the means available to interact with it, but also of the meanings that are associated with the activity itself (Leontiev, 1978).

Here, another question is raised, what is the relationship between completing a task and learning mathematics? In our view, tasks are proposed to motivate learners to engage with practices associated with the set of artefacts that have, historically and

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culturally, come to represent the body of knowledge we call mathematics. In the socio-cultural perspective which informs this view, learning can be defined as participating in, and appropriating, these practices. Using the concept of activity defended by Leontiev (1978), appropriation is a social process in which participants aim to make their own objects already steeped in cultural meaning. The process of appropriation occurs, necessarily in the case of mathematics, on the basis of actions mediated by semiotic systems. Mathematical activity, then, occurs as a dialectical process, in which individuals interact with the environment and with other individuals to attribute sense to aspects of the knowledge and experiences developed in the course of human history. As a result of this activity, objects of the environment, recognised by the senses, acquire the character of objects of reflection (Fernandes, 2008, p.47). This brings us to another aspect central to the theoretical framework that guides our approach to task design: the role of the senses in mathematical cognition. We see links between the socio-cultural perspective developed by Soviet psychologists in the last century with the views of researchers in the area of embodied cognition today, who argue that our mathematical understandings, like all others, are structured by our encounters and interactions with the worlds we experience via our bodies and our brains (Gallese and Lakoff, 2005). Indeed, Radford (2006) argues that the body itself can serve as a semiotic system, through acts of perception, gestures and other movements. Other semiotic systems for Radford include artefacts, language and signs. Jointly, then these represent the tools through which task demands are to be mediated.

With this view of mathematics learning in mind, we return to the question of designing tasks, tools and teaching interventions for inclusive learning scenarios. More specifically, in this article, we concentrate on learning scenarios in which students who lack access to one or other sensory field act; that is, we focus on learners who are deaf and learners who are blind. To enable the participation of students with sensory limitations, it is important to understand how the different channels through which they experience the world mediate the processes by which they appropriate mathematical knowledge, and to recognise that the mathematical practices that are depend on these experiences. Once again, we draw here from the work of the Vygotsky and his colleagues in the former Soviet Union and particular to work in the area of what at the time was called *Defectology*. Vygotsky (1997) proposed an approach to understanding the learning of students with sensory, motor or cognitive disabilities which involves considering how and when the substitution of one (non-functioning) tool by another may engender different forms of activity (Healy & Fernandes, 2011; Healy & Powell, in press). His approach stressed the potential for development of learners with disabilities, rather than positioning them as deficit in relation to some supposed "norm".

> "The positive particularity of a child with a disability is created not by the failure of one or other function observed in a normal child but by the new structures which result from this absence […]. The blind or deaf child can achieve the same level of development as the normal child, but through *a different mode, a distinct path, by other means*. And for the pedagogue, it is particularly important to know the uniqueness of the path along with the child should be led" (Vygotsky, 1997; p.17 – emphasis in the original)

In relation to empowering those without access to one or other sensory field to participate in social (cultural) activities, for Vygotsky, the solution lies in seeking ways to substitute the traditional means of interacting with information and knowledge with another. For example, he suggested that the eye and speech are "instruments" to see and to think respectively, and that other instruments might be sought to substitute the function of sensory organs (Vygotsky, 1997). As mathematics educators, we interpret this message from Vygosky as implying we need to pay attention to – and where necessary create – a multitude of (substitute) semiotic systems to mediate mathematics learning.

Our approach to design

The research strategy we adopt is based on the establishing of partnerships between school- and university-based participants – researchers, school teachers and school students – who collaborate on the design of tasks and tools for use in the mathematics classrooms of the teachers and students. In these partnerships, participants work together to conduct a process of co-generative inquiry (Greenwood & Levin, 2000), a kind of participatory action research in which all participants co-generate knowledge through a process of collaborative communication.

The process of designing learning activities is not a simple one and passes through a number of stages. Not all the participants are necessarily involved in all the stages, although usually at least one member from the school and one from the university is present in each. The first stage involves designers in identifying particular challenges associated with the learning of the mathematical topic in question and in developing and testing hypotheses about how best to engender the intended learning. The topics selected are those that are emphasised in the mathematics curriculum that the schools are following and the starting point for the design process is twofold, aiming to combine both pragmatic and theoretical concerns. On the one hand, the teachers and sometimes also the students themselves, brings examples of particular difficulties and problems they have experienced. At the same time, we also consult the existing literature to attempt to determine what previous research tells us about students' conceptions in the chosen topic. On the whole, we have found relatively little research addressing the mathematical learning processes of either blind students or deaf students with respect to the majority of topic areas we have addressed. This means that the first versions of the task are often developed on the basis of what we know about sighted and hearing learners and hence may not be fine-turned to the particular strengths of those who do not see with their eyes or who do not speak with their mouths.

This is one of the reasons that we believe it is crucial to involve the students, as well as teachers and researchers in the design process. Student participation occurs early in the design process, as students are invited, either individually or in small groups, to work on the first prototypes of the tasks and tools under development. For these first tests, we have tended to work exclusively either with blind students or with deaf students. The meetings are videotaped and represent a means for reviewing our developing theoretical models and revising our hypotheses so that they can be operationalised given the particularities of the schools and students involved. Our tendency has been to develop tools and task sequences simultaneously and to modify both as the sequence is applied in practice during these tests. It is only after the scenarios have been tested and analysed, that we consider re-enacting the scenarios in teacher's mathematics classrooms.

Examples of the design process in practice

To illustrate this process in a little more detail, we have selected two examples from the collection of learning scenarios we have investigated: one in which tasks were to be mediated using material objects and a second involving the use of a tactile, digitally-controlled tool designed to permit the exploration of graphs of polynomial functions.

The first example involves the topic Matrices, a topic that is introduced to students in the second year of High School (that is the 11^{th} year of compulsory schooling) in the curriculum currently followed in the state of São Paulo. One motivation for focussing on this topic comes from the comments of deaf students in one of our partnership schools and blind students in another. The deaf students, for example, described Matrices as "*something that has brackets and numbers*", but were not confident in manipulating paper-based representations these objects. Their teacher, fluent in Libras (the Brazilian sign language), suggested that the lack of specific signs for the vocabulary associated with matrices served as a complicating factor in teaching the topic. The blind students, too, spoke of their difficulties in solving tasks with matrices, which they described as "drawings with numbers inside". We found no research studies which addressed interactions of either blind or deaf students with matrix representations. We decided to attempt to construct a form of representing matrices which would permit both deaf students and blind students to construct them and operate upon them (more details of the design process are available in Silva, G. G., 2012). The tool MATRIZMAT is a very simple one, made up of plastic boxes (5cm by 5cm by 3cm) which could be joined to each other by magnets fixed to each of the boxes' four sides (Figures 1 and 2). In the version for deaf students, the numbers written on foam-rubber rectangles could be placed in the cells of the matrices (Figure 1), whereas, for the blind students, we made use of the lids of the boxes, with numbers in Braille stuck onto the top (Figure 2).

Figure 1: MATRIZMAT with written numerals

Figure 2: MATRIZMAT with numbers in Braille

The tasks for both student groups had the aim of introducing the language associated with matrices, their organisation in rows and columns, determination of the order of a given matrix, identification of equal matrices and matrix addition. It is not our intention in this article to present in detail the interactions of the students with these tasks, but perhaps it is worth stressing that the material tools enabled to both student groups to develop efficient ways of expressing matrix structure. Figure 3, for example, shows Maria using the signs developed by the students themselves to indicate position $\mathbf{a}_{12}$ of a 3 by 3 matrix**.** It seemed that the layout of the matrix in a physical, palpable form helped the group to develop ways of talking about its structure – something that they had had difficulty to do when operating with the paper and pencil representation.

Figure 3: Maria signing position $\mathbf{a}_{12}$ of a 3 by 3 matrix

In the case of the blind students, the importance of the tactile tool was most evident when they were comparing or adding two matrices (see, for example, Figure 4). Being able to explore spatially the positions of the elements of the matrix enabled them to experience matrices in ways that correspond to those of their sighted contemporaries. This was rather different to their previous experiences, which had involved Braille representations in which matrices were presented in a form that did not emphasise the spatial layout of the elements and in which they had found it very difficult to locate the elements in different matrices which should be added to each other.

Figure 4: Adding matrices of order 3x2

The second example evolved as we attempted to develop tasks related to polynomial functions that would be accessible to blind students. We knew from the students themselves that tasks involving graphical representations of functions were something that their teachers tended to avoid assigning (Silva, B. J., 2012). From the research literature about (sighted) students' conceptions of functions, we conjectured that a tool in which blind students could experience not only static representations of the locations of particular points on the Cartesian plane or static representations of the graphs of specific functions, but could also feel the graphs of different functions appear as the independent variable changed, might afford more dynamic views of function and help them understand the dependence relationship that exists between the independent and dependent variables. The tool that we designed to permit blind student to engage in such tasks was composed of a digitally controlled board made up of a rectangular matrix of pins, each of which represented a point on the plane. When particular point is requested or a graph of a given function plotted, the relevant pins are raised up (sequentially as the value of the independent variable increases in the case of the graph of a function), allowing the student to feel the image as it is produced. Figure 5 presents the final version of the tool, while Figure 6 shows a blind user, Alice, as she feels the graph of a function as it reveals itself to her.

Figure 5:The current version of the tactile graphing tool and its digital interface

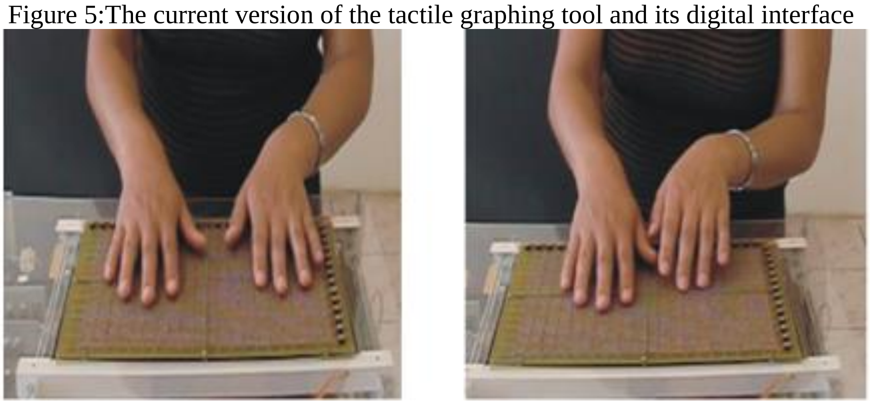

Figure 6: Alice feeling a graph as it is plotted

To date, we have only tested this tool with students who do not see with their eyes. Of course, it could also be used by the sighted, but perhaps represents a rather expensive option to the on screen dynamic graphing tools which already exist. There is a question though: are the experiences of seeing a graph as it emerges in front of one's eyes and feeling it emerge with one's hands cognitively identical? Our conjecture is that they may not be – the act of moving one's hands to find points in ways not guided by one's eyes seems rather different to having a kind of global access to the whole plane upon which the graph appears. The difference in the strategies afforded by these different ways of perceiving and the properties of the graphs emphasized in these two conditions is something we believe merits further research.

Reflections on the relationships between task and tool design

The theoretical influences that inform our approach to design indicate that the processes of creating tasks and the tools by which they are to be mediated are best tackled simultaneously. They also lead us to recognize, as Cole e Wertsch (1996, p.255) have pointed out, that the insertion of tools into situations with instructional intent does not simply serve to facilitate the mental processes that occur within them, it fundamentally forms and transforms these mental processes, conditioning the practices of the learners who operate the tools to the particular practices associated with their use. Moreover, it is not only the learners whose practices are transformed: the introduction of any artefact into a given situation might offer new – and even more efficient means – to resolve a problem, but it also changes the very nature of the task (Béguin & Rabardel, 2000, p.2). In this paper, we have concentrated on the process of design and not on the resulting interactions of those who participate in the learning situations into which the resulting tasks and tools are incorporated, but we believe that the particularities of the students with whom we work help to illustrate the extent to which it is not only the material and semiotic tools we provide that impact upon the practices which emerge in the scenarios. Equally important are the bodily resources through which tool and task are experienced, with different sensory-motor systems potentially affording different modes of acting mathematically and, hence, different paths by which mathematical meanings might be appropriated.

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