

# Assistance of students with mathematical learning difficulties: how can research support practice?

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**Abstract** When looking at teaching and learning processes in mathematics education students with mathematical learning difficulties or disabilities are of great interest. To approach the question of how research can support practice to assist these students one has to clarify the group or groups of students that we are talking about. The following contribution firstly concentrates on the problem of labelling the group of students having mathematical difficulties as there does not exist a single definition. This problem might be put down to the different roots of mathematics education on the one hand and special education on the other hand. Research results with respect to concepts and models for instruction are multifaceted based on the specific content and mathematical topics as well as the underlying view of mathematics. Taking into account inclusive education, a closer orientation to mathematical education can be identified and the potential of

selected teaching and learning concepts can be illustrated. Beyond this, the role of the teacher, their attitudes and beliefs and the corresponding teacher education programs are discussed.

**Keywords** Mathematical learning difficulties · Inclusive education · Special education · Teacher education

## 1 Introduction: mathematics learning, special education and inclusion—setting the scene

The following paper reports part of the work of the survey team “Assistance of students with mathematical learning difficulties—How can research support practice?” for ICME 13. When starting the work the important aspects of defining students with mathematical learning difficulties, the role of teachers and teacher education programs as well as effective teaching programs and concepts came into the focus. Looking back to the ICME conferences of the last 20 years we identified the widespread contributions in the corresponding topic study groups or discussion groups. It became obvious that we have to take into account different disciplines: Although mathematics should be in the centre, special education, psychology and pedagogy also play important roles. One problem is that the different fields might follow different paradigms, which in turn might lead to contradictory conclusions with regard to the teaching and learning of students with mathematical learning difficulties. As a consequence, we will not cover the whole range of impairments with regard to learning mathematics. Moreover, we have chosen to discuss the fields of teacher education and concepts of effective teaching and learning by focussing on exemplary cases. The paper aims therefore

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- to describe definitions of mathematical learning difficulties and the problem of labelling,
- to discuss findings related to effective teaching practices and intervention strategies,
- to discuss concepts of assistance in the context of inclusive education,
- to draw conclusions for teacher development.

In the title of this paper the term “students with mathematical learning difficulties” has been chosen to point to a group of learners perceived as being in particular need of assistance. But who is included in this group? This question is complicated by the fact that different terms are applied to describe learners who compose the target groups of (special) education in different countries<sup>1</sup> and at different points in history. In the first instance, we might interpret the term “students with mathematical learning difficulties” to be synonymous with terms such as “students with mathematical disabilities” or “students with special needs in relation to mathematics”, but a closer look at the terms and the contexts in which they are used reveals that they may be associated with different approaches to teaching and learning, to different models of disability and to whether disability—or difficulty in learning mathematics, in our case—is seen essentially as an individual attribute or as a consequence of barriers imposed by society. Our paper is organized as follows: First, we discuss different definitions and assumptions concerning mathematical learning difficulties or disabilities. In Sect. 2, we present results of selected intervention studies, followed by reflection upon the views of mathematics and mathematics learning that underpin these studies. In the third section, we concentrate on more qualitative approaches to research involving learners identified as under-achieving mathematically and the complex nature of the challenges associated with improving their relationships with mathematics. In this section, exemplary approaches for teaching and learning settings are illustrated. Section 4 focuses on the teacher’s role and beliefs and on more general aspects of teacher education programs

<sup>1</sup> Since this text is produced in English, it concentrates on the different terms used in contexts mediated by this language. In fact, the issue of how disabled people or people considered as having learning difficulties are labelled is far more complex than a paper in the English language can express. In Brazil, for example, where the dominant language is Portuguese, the term “people with disabilities” is translated as “pessoas com deficiências”, which a literal translation back to English would read as “people with deficiencies”, a term that many would reject for implying a deficit-model of difference. In Quebec (where live a francophone minority in the Anglophone North America) the expression At-Risk Students is preferred to the expression “learning difficulties”, a choice which has contributed to a large decrease in the number of specialized classes (Mels 2007). In Gabon, specialized classes do not exist and the label “students failing at school” is used.

relevant to students with mathematical learning difficulties, followed by conclusions and perspectives for future research in Sect. 5.

## 2 Mathematical learning difficulties: definitions and usage

In this section we examine and critique definitions and usage of the term “mathematical learning difficulties” and related terms.

### 2.1 Mathematics learning difficulties and the problem of labelling

The terms “learning difficulties” and “learning disabilities” and their use in different countries and different contexts require some clarification in order to explore their influence on the development and use of diagnostic criteria and instruments, especially because the ways in which students with mathematics learning difficulties are described carry implicit meanings of both “learning difficulties” and “mathematics”. Differences in these meanings, with regards both to which factors contribute to learning difficulties and to how the nature of mathematics is understood, underpin tensions- and commonalities- evident in the literature coming from the domain of special education as compared to mathematics education as noted by Boyd and Bargerhuff (2009).

Before considering the process of labelling, it is useful to comment briefly on the variation in the actual labels used, even in countries sharing the same language. Education services in the UK, for example, use the term “learning difficulty” to refer to children and young people who have “specific learning difficulties”, but do not have a general impairment of intelligence, other countries like the US, Canada and Australia use the term “learning disability” for the same group. In this text, the term “learning difficulty” is employed except when citing studies in which the authors use another term.

### 2.2 Definitions and frameworks

While some researchers would argue that low achievement is a social construct, “not a fact but a human interpretation of relations between the individual and the environment” (Magne 2003, p. 9), others seek to attribute low achievement to the presence of cognitive disorders or a discrepancy to IQ. According to an extensive review of the literature on special educational needs in mathematics in 2003, medical models, which position difficulties as innate, were adopted in the majority of studies surveyed (Magne 2003). Magne claimed that, at that time, the move toward social and cultural interpretations was only beginning to emerge in this particular area of research. Socio-political

approaches recognize that “the constructs of both ability and disability are socially, culturally and politically constructed facets of identity and experience” (Broderick et al. 2012, p. 1) and locate disability in the oppressive practices to which those whose bodies are perceived to deviate from what is revered as normal are subjected (Shakespeare and Watson 2001; see also Campbell 2001). This form of oppression has been termed ableism: “a network of beliefs, processes and practices that produces a particular kind of self and body (the corporeal standard) that is projected as the perfect, species-typical and therefore essential and fully human. Disability then, is cast as a diminished state of being human” (Campbell 2001, p. 44). Ableism, and how it acts to exclude and disable mathematics learners is currently an under-researched area in mathematics education, although there are signs that this might be changing, with growing numbers of researchers investigating how ableist assumptions about what constitutes the normal body contribute to the marginalisation of those whose cognitive, emotional, physical and or sensory configurations differ from what is currently defined as socially desirable (some examples include Borgioli 2008; Healy and Powell 2013; Marcone and Atweh 2015; D’Souza 2015; Healy 2015). Research in this direction suggests that rather than being the consequence of internal, individual factors, students’ underperformance in mathematics can result from “their explicit or implicit exclusion from the type of mathematics learning and teaching environment required to maximize their potential and enable them to thrive mathematically” (Gervasoni and Lindenskov 2011, p. 308).

In order to emphasise the social shaping of learners’ identities and experiences, Bagger and Roos (2014) used the term “students in need of special education in mathematics” rather than the more commonly used term “students with special needs” (chosen, for example, in naming the topic study group in this area at the International Congress on Mathematical Education conferences (ICME) that have occurred this century). This brings us back to the question of which students compose the group of interest for this survey. Gervasoni and Lindenskov (2011) draw attention to the same challenge (they preferred to use the term “special rights for mathematics education”), highlighting two groups. The first group encompasses learners with disabilities defined by the United Nations (UN) convention on the rights of persons with disabilities, as having long-term physical, mental, intellectual or sensory impairments which in interaction with various barriers may hinder their full and effective participation in society on an equal basis with others (UN 2006). The second group they describe as being composed of those who underperform in mathematics. Deciding and defining who should be classified as a member of this second group raises a multitude of questions for those interested in issues of equity and

social justice, since it involves conditions that tend not to have known organic causes and are defined on the basis of psychometrical tests, or other measures in which learners’ behaviours or responses are deemed to deviate from established benchmarks or norms—as for example in the recent Response to Intervention (RTI) model (Fuchs and Fuchs 2006).

In this paper, we focus mainly on students from this second group, since it should not be assumed that all disabled students will necessarily have difficulties in learning mathematics. Nonetheless, it is perhaps worth mentioning briefly that research into the mathematical practices of students in the first group reinforces the argument that mathematical performance is not determined by individual attributes alone but also by the affordances and constraints they encounter in mathematical learning environments, and indicates that approaches built on the premise that all students learn mathematics in the same ways is likely to disable particular groups of learners (for more information, see for example, Healy and Fernandes 2011, 2014; Marschark et al. 2011; Nunes 2004; Bull 2008; Nunes and Moreno 2002; Pagliaro 2006; Marschark and Hauser 2008).

Returning to the second group of learners and the ways in which students come to be identified as having specific difficulties in learning mathematics, it is important to stress from the outset that this is an area of some controversy. This is especially so because, when the question of diagnosis is at the forefront, it is medical models and models which posit achievement as something inherent to the individual which tend to dominate. For example, according to 10th International Classification of Diseases (ICD 10, WHO 2016), amongst the entries associated with specific developmental disorders of scholastic skills, is the category specific disorder of arithmetical skills (F81.2). This disorder is described as a “specific impairment in arithmetical skills that is not solely explicable on the basis of general mental retardation or of inadequate schooling. The deficit concerns mastery of basic computational skills of addition, subtraction, multiplication, and division rather than of the more abstract mathematical skills involved in algebra, trigonometry, geometry, or calculus” (ICD 10, WHO 2016).

This definition is used as a basis for the widespread, if heavily criticised, use of the IQ-discrepancy model, where a mathematical learning disorder is diagnosed as a result of a discrepancy between IQ and mathematics performance level. Amongst the various problems associated with this model, Francis et al. (2005) stressed that it can lead to over-identification at upper levels of IQ and the under-identification at lower levels of IQ. They also describe how this model leaves unspecified the point at which a discrepancy becomes significant. Another issue is whether or not the differences in IQ levels, or indeed other such measures of performance, permit the identification of the particular

characteristics of different students groups (Murphy et al. 2007).

In the light of such criticisms, another influential classification system, the Diagnostic and Statistical Manual of Mental Disorders (DSM V) published by the American Psychiatry Association (APA), no longer makes use of the discrepancy model. In previous versions, what was called a mathematics disorder was listed, however, in DSM V, this has been redefined as one of the subtypes of a “specific learning disorder”, that is, a neurodevelopmental disorder that impedes the ability to learn or use specific academic skills. The symptoms are described as follows:

Difficulties mastering number sense, number facts, or calculation (e.g., has poor understanding of numbers, their magnitude, and relationships; counts on fingers to add single-digit numbers instead of recalling the math fact as peers do; gets lost in the midst of arithmetic computation and may switch procedures). Difficulties with mathematical reasoning (e.g., has severe difficulty applying mathematical concepts, facts, or procedures to solve quantitative problems). (DSM V 2016)

Further important criteria included in the DSM V are skills that are substantially and quantifiably below those expected for the individual’s chronological age, and that cause significant interference with academic or occupational performance, or with activities of daily living. In addition, learning difficulties begin during school-age years and must be persistent and specific.

The changes from the DSM IV to the DSM V definitions of learning disorders reflect the lack of consensus as to the precise nature of the so-called specific learning disorders and the problems that arise when learning difficulties in mathematics are treated using exclusively neuropsychological perspectives. Healy and Powell (2013) reviewed some of the critiques:

- the lack of a robust consensus as to its defining characteristics and diagnostic criteria, except that it involves poor recall of number facts (Gifford 2005; Mazocco and Myers 2003),
- over-emphasis on the use of tests involving standard calculation procedures considered to be “normal” (Gifford 2005; Ellemor-Collins and Wright 2007),
- unsubstantiated assumptions that all students learn in the same way (Ginsburg 1997) and that both learning difficulties and responses to teaching interventions can be expected to be homogenous (Dowker 1998, 2004, 2005, 2007),
- failure to recognize the multitude of environmental and socio-emotional factors that interact with cognitive and neural aspects to contribute to the heterogeneity of mathematics difficulties (Kaufmann et al. 2013).

Nonetheless, despite this lack of consensus, many researchers agree that it is likely that differences exist between individuals in the neuroanatomical and neurological processing of number even if these factors do not act in isolation from environmental ones in students’ mathematical performance. Moreover, despite the tendency in some studies to emphasize procedures over concepts, specific difficulties at both the procedural and conceptual levels have been identified as typical amongst those who have mathematical learning difficulties. Procedural problems occur in relation to fact retrieval and lead to the persistent use of (finger) counting strategies for easy computation problems (Andersson 2008; Hanich et al. 2001; Häsel-Weide et al. 2013; Ostad 1999). Difficulties associated with conceptual understanding are often apparent in the domains of appropriating different aspects of the place value system like grouping, degrouping and understanding place value (Mazzocco et al. 2008; Moeller et al. 2011; Moser Opitz 2013; Vukovic and Siegel 2010; Scherer 2014). Other researchers have reported problems with verbal counting, especially counting (by groups) (Moser Opitz 2015; Schäfer 2005; Scherer 2014) or understanding counting principles (e.g. Geary 2004). In addition, the transformation of word problems into mathematical expressions and, as a consequence, difficulties in solving word problems seem to be widespread (e.g. Montague and Applegate 2000; Moser Opitz 2013; Parmar et al. 1996; Zhang and Xin 2012).

Not all of the mathematical learning difficulties cited in the previous paragraph are contemplated in the ICD 10 and DSM V definitions, which reside in the medical rather than educational community. Indeed, recent publications of the Eurydice Network<sup>2</sup> use a broader concept of those with mathematical difficulties, using the term to refer to any group of students with low achievement in mathematics:

Low achievement is the situation where a child fails to acquire basic skills while they do not have any identified disability and have cognitive skills within the normal range. In those cases, low achievement may be considered as a failure of the education system. (European Commission, n.d., p. 4)

The problem with this definition is that it implies the issue of identifying disabilities related to the learning of mathematics has been resolved. It also involves the use of some instrument to determine what counts as the normal range.<sup>3</sup> To a certain extent, this definition can be compared to the ICD 10 definition, except that “identified disability” has substituted “inadequate schooling” (another not easily

<sup>2</sup> Network on education systems and policies in Europe [http://eacea.ec.europa.eu/education/eurydice/index\\_en.php](http://eacea.ec.europa.eu/education/eurydice/index_en.php).

<sup>3</sup> The cited report privileges the levels established in the OECD Programme for International Student Assessment (PISA).

definable or measurable term). Another potential problem is that, in the case of those with an identified disability, the Eurydice approach to low achievement might lead to interpretations in which the education system is, to some extent, excused from tackling their mathematical difficulties. This is highly questionable: even those who argue that dyscalculia is a core, brain-based, deficit in number processing, acknowledge the possibilities for developmental interaction between the brain and experience, leading Butterworth et al. (2011) to suggest that:

one way of thinking about dyscalculia is that the typical school environment does not provide the right kind of experiences to enable the dyscalculic brain to develop normally to learn arithmetic. (p. 1050)

Like the Eurydice approach, the message here is that, regardless of the causes, it is important to offer students with mathematical learning difficulties environments that enable them to thrive mathematically. How then might the teaching community intervene in ways that enable students to negotiate the difficulties they experience? To explore this question, the next section focuses on the results of studies into interventions aimed at improving the performance of students with mathematical learning difficulties.

### 3 What do we know about effective teaching practices in mathematics classrooms? Intervention studies

In this section we consider the findings of interventions aimed at improving the mathematics achievement of students with mathematical learning difficulties. We first review the results from meta-analyses and then consider the findings of particular studies at various levels of schooling before considering the complex conditions surrounding special education teaching. One has to take into account that the conditions in schools and classrooms, and indeed the structure of school systems, differ in the various countries, especially when speaking about students with mathematical learning difficulties. Nevertheless, results from different countries are used for structural and political decisions, and our aim is to give a critical review that could help to better value and classify some of the studies.

#### 3.1 Effective interventions: results of meta-analyses

Meta-analyses and literature overviews provide information on effective intervention practices for students with mathematical learning difficulties. However, keeping in mind the lack of a generally accepted definition of mathematical learning difficulties, it is important to recognise that different studies have investigated different samples (see

also, Murphy et al. 2007). Moreover, one has to take into account which mathematical topics and competencies have been investigated. It is, therefore, difficult to compare the results of the studies and some of the findings seem contradictory. In addition, many studies focus on the improvement of single competencies.

Nevertheless, most analyses show that direct or guided instruction seems to be fruitful for students with mathematical learning difficulties. A meta-analysis by Kroesbergen and Van Luit (2003) found that the majority of the included studies described interventions in the domain of basic arithmetic skills. Direct instruction and self-instruction (e.g. self-regulated strategies) were found to be effective. Direct instruction appears to be effective with regard to basic mathematical facts and self-instruction with regard to problem-solving. Interventions involving the use of computer-assisted instruction and peer tutoring showed smaller effects than interventions in which the teacher instructed the students. According to a meta-analysis from Gersten et al. (2009), explicit instruction and teaching students to use heuristics led to practically and statistically important increases in effect size. The instructional components that appear to be fruitful are:

1. teaching heuristics to solve word problems;
2. explicit instruction;
3. the use of graphical representations and manipulatives;
4. thoughtful selection and sequencing of instructional examples, and finally;
5. encouraging students to verbalize their own strategies or the strategies modelled by the teacher.

A meta-analysis by Ise et al. (2012) of studies from German-speaking countries showed one-to-one training to have advantages over small group interventions, computer-based programs, and interventions integrated into the classroom. In addition, the duration and intensity of the program, and the qualifications of the teachers proved to be important factors. Zhang and Xin (2012) carried out a meta-analysis of interventions concentrating on word problem-solving problems in mathematics. The most effective intervention was determined to be a technique which focused on the representation of the structure of the word problem. What they term as “cognitive strategy training” was next in terms of effectiveness, followed by the strategies involving assistive technology.

#### 3.2 Looking at different school levels

Although a remarkable number of studies have focused on mathematics education in pre-school and kindergarten levels, in the following we concentrate on primary and

secondary school levels, because difficulties with learning mathematics manifest during the school years.

At the primary level studies have pursued different objectives. Some have examined the impact of training for procedural competencies (e.g., fact retrieval), whereas other studies have stressed conceptual understanding. Fuchs et al. (e.g., 2009, 2010, 2014) have carried out several studies to examine the impact of training programs on procedural competencies. Their study of Grade 3 students emphasized fact retrieval (working on number combinations with counting strategies and flash cards) and word problems (Fuchs et al. 2009). The latter involved the step-by-step introduction of different kinds of word problems (change, combine, compare, and equalize relationships). Compared to a control group, a significant effect was found for fact retrieval, but not for problem solving. In another study, an intervention of strategic counting and deliberate practice was successful with regard to number combination skills (Fuchs et al. 2010). Fuchs et al. (2014) found in a study of Grade 2 students comparing interventions focussed on calculation and word problem solving that the calculation intervention improved calculation but not word problem solving outcomes. The word problem intervention enhanced word-problem but not calculation outcomes. In addition, the word-problem intervention provided a stronger route than the calculation intervention to pre-algebraic knowledge.

Despite the significant results associated with training for fact retrieval with counting strategies in the studies of Fuchs et al. (2009, 2010), some critical questions arise. First, long-term effects have not been investigated in this research. Second, no information is available as to whether the students actually improved with fact retrieval, or simply used counting strategies more quickly. According to several authors (e.g., Andersson 2008; Gaidoschik 2012; Moser Opitz 2013) persistent finger counting is an important characteristic of learning difficulties in mathematics. Moser Opitz and Ramseier (2012) showed that strategy use (counting strategies versus fact retrieval) should be explicitly incorporated in diagnostic instruments.

Andersson (2010) underscored the importance of fostering the domains of conceptual knowledge (e.g., place value, base-ten system, relationship within and between arithmetic operations), procedural knowledge (i.e., knowledge of calculation strategies, flexible use), factual knowledge (e.g., arithmetic facts), and skills for solving word problems. Studies that have focussed on the improvement of conceptual understanding, sometimes combined with procedural training, include that of Ennemoser and Krajewski (2007) who found a significant effect on mathematics achievement of an intervention on the part-whole relationship in Grade 2. Pedrotty Bryant et al. (2008) evaluated also a program with first and second graders which emphasized

conceptual understanding in relation to number concepts, the base-ten number system, place value, and addition and subtraction combinations. The intervention was successful in Grade 2, but not in Grade 1. The authors argued that the intervention period may have been too short for first graders. A similar program was successfully carried out with a German sample (Wißmann et al. 2013) and the students in the intervention group had significantly greater learning gains compared to a control group. Pfister et al. (2015a, b) conducted a study, involving two intervention groups and a control group to examine how teachers and special education teachers implemented a remedial mathematics program in a classroom setting, focusing on conceptual understanding (place value, meanings of operations), selected procedural skills (e.g., automation of number combinations, counting by steps), and adaptive teaching practices. Both interventions groups used the same material, whereas one of the groups had an extra in-service training ( $2 \times 3$  h). Surprisingly, a significant effect was found only for the intervention group without an extra in-service training. Aspects of class composition (e.g., the number of second language learners per class) or different general conditions in classroom may have led to this result.

At the secondary level, only a few studies are available. According to a survey by Maccini et al. (2007), the most frequently used type of intervention at this level is direct instruction and “drill and practice” teaching. Many studies have focused on higher mathematical domains like algebraic skills and concepts and have not covered basic mathematical knowledge from primary school. However, as emphasized by Ennemoser et al. (2011), Moser Opitz (2013) and Scherer (2014), low achievers in higher grades lack basic competencies such as counting in groups or understanding the base-ten system, even with small numbers. This suggests that remedial intervention programs aimed at fostering basic arithmetic understanding may also be important for secondary school students.

Woodward and Brown (2006) reported having implemented a middle school program emphasizing conceptual understanding of primary arithmetic and problem solving in the classroom which led to significantly higher learning gains for the intervention groups. Based on these results, a longitudinal study of Freeseemann (2014; Moser Opitz et al. accepted) involving lower secondary school students focused on an intervention with regard to conceptual understanding of basic arithmetic: the central ideas of the base-ten number system (grouping, degrouping, place value, the meaning of the operations in terms of building up a “mental model”, and selected procedural skills (e.g., adding up to 100, counting by groups). The intervention groups outperformed a control group with regard to the content covered in the intervention, including in the follow-up 4 months after the intervention. Interventions on the development of

conceptual thinking seems to be fruitful at both primary and secondary level.

### 3.3 Reflections on the intervention studies

The results of these studies raise questions. First, it is not always clear what is meant by “guided instruction” or “explicit instruction”. Whilst some authors emphasize support strategies which include the use of graphical representations and manipulatives, thoughtful selection and sequencing of instructional examples, and encouraging students to verbalize their own strategies or the strategies modelled by the teacher (Gersten et al. 2009), all of which aim to foster conceptual understanding, others understand explicit instruction in a narrow way.

Second, some of the studies focus on a narrow perspective of the discipline, in which learning mathematics is seen as skill acquisition and success in mathematics is about obtaining correct answers. Such views do not reflect the broader understandings of mathematics encompassed in most curricula that include content domains other than number e.g., geometry, statistics and probability, algebra, and measurement. Most importantly, such curricula typically include process or mathematical thinking strands (e.g., problem solving, reasoning, development of individual strategies) that portray mathematics as a way of thinking and making sense of the world and that is arguably closer to mathematicians’ own views of their discipline (Burton 2002). Such views of mathematics are rarely reflected in the studies which focus on procedural skills alone.

They are however addressed in studies which emphasise conceptual understanding. (e.g., Moser Opitz et al. 2016; Pedrotty Bryant et al. 2008; Pfister et al. 2015a; Woodward and Brown 2006). Essentially, the interventions reported do not cover the whole range of mathematical domains, but focus on topics that are known as “stumbling blocks” for many students with learning difficulties in mathematics. This leads to a further challenge. While it may be the case that some difficulties are more common than others—it is known, for example, that students with persistent mathematical learning difficulties often have problems with verbal counting, understanding place value and understanding basic operations (e.g., Ennemoser et al. 2011; Moser Opitz 2013; Vukovic and Siegel 2010; Scherer 2014) making it difficult for these students to acquire further arithmetical knowledge. On the other hand, it is also essential to realize that:

No two children with arithmetical difficulties are the same. It is important to find out what specific strengths and weaknesses an individual child has; and to investigate particular misconceptions and incorrect

strategies that they may have. Interventions should ideally be targeted toward an individual child’s particular difficulties. If they are so targeted, then most children may not need very intensive interventions. (Dowker 2004, p. 45)

This viewpoint was echoed by Gervasoni and Sullivan (2007) whose research with low achieving mathematics learners revealed that learners who have difficulty in one aspect of number learning do not necessarily have difficulties in all areas, and led them to conclude that “there is no single ‘formula’ for describing students who have difficulty learning arithmetic or for describing the instructional needs of this diverse student group” (p. 49). Developing interventions for students with learning difficulties in mathematics is, therefore, a “balancing act” between giving guidance and taking into account the learners strategies and concepts; and focussing on well known “stumbling blocks” without forgetting that mathematics means more than arithmetic (see also Sect. 3).

A third issue is that in most of the intervention studies presented in this section, achievement is treated as an individual rather than a social construct. Researchers adopting critical socio-political perspectives on special education have raised grave concerns that the use of individual measures to diagnose a social construct has led to the disproportionate representation of ethnic minority students, indigenous students groups and those living in poverty in the psychometrically defined, or judgemental, categories of those eligible for special education (Artiles et al. 2006; Dyson and Gallannaugh 2008; Mantoan 2009; McDermott 1993). Though not limited to mathematics education, this disproportionality is an indication of how practices based on the determination of “normal” or “ideal” achievement, and the positioning of those that deviate from this norm as problematic and in need of remediation, can result in the legitimization of the marginalization of those whose learning trajectories deviate from what is considered normal (Healy and Powell 2013). For Ferri (2012), the marginalization of students with learning difficulties results from the assumption that students deemed eligible for special educational are classified as fundamentally different from their non-disabled peers (p. 863), that is, retained in each successive educational reform is the idea that there are two distinct student types, the “typical” or “normal” and the “disabled” or “abnormal”. To demonstrate this point, Ferri analyses discourse associated with some of the recent intervention programmes in the US, pointing to phrases such as “If students respond to the treatment trial, they are seen as remediated and disability-free and are returned to the classroom for instruction” (Fuchs and Fuchs 2006, quoted in Ferri 2012, p. 870). Social models of disability offer an alternative to creating this binary opposition between

special student/normal student, arguing that disability is not something that resides in the individual but results from the interaction between the individual and their environment. In the social view, disability is a process that happens when one group of people create barriers by designing a world only for their way of living- or learning. It is such social models that underpin the move towards inclusive education and the commitment to the dismantling of barriers to the full participation of all learners. In the next section, we consider the challenges associated with inclusive approaches to mathematics education.

## 4 Inclusive education

The UNESCO International Bureau of Education (2009) defined inclusive education as “an ongoing process aimed at offering quality education for all while respecting diversity and the differing needs and abilities, characteristics and learning expectations of students and communities, eliminating all forms of discrimination” (p. 18). This definition is at odds with the view that only remediated learners have a place in mainstream classrooms, since, in inclusive schools, difference is seen as a factor that enriches the educational process and not as a deficiency that impedes learning or justifies segregation. Inclusive schools aim to involve all learners in quality learning experiences which empower them to become active participants in a more equitable system. The movement for the development of inclusive education systems is concerned with eliminating all forms of discrimination and marginalisation, but it has come to hold a special significance in relation to disabled students and students with learning difficulties because it has been accompanied in many countries by a move of these student groups from specialised to mainstream settings.

As with the terms “learning disabilities in mathematics” and “mathematical learning difficulties” there does not exist a common understanding of the term “inclusive education” (Ainscow 2013) and there are many possible ways of viewing the notion of inclusion. For example, conceptions are likely to be mediated by factors such as the organisation of the school system (differentiated or comprehensive), legal regulations related to the provisions for students eligible for special education, policies related to the progression between grades (exam-based or age-based) and practices related to the organisation of classes (mixed-ability or streaming). Skovsmose (2015) has argued that inclusion represents an example of what he calls a contested concept, a concept that can be given different interpretations that operate in different ways in different discourses. For him, contested concepts represent controversies that can be of a profound political and cultural nature. His view of inclusion is one that rejects the idea of bringing learners

into some (politically) presumed “normality”. “Instead inclusive education comes to refer to new forms of providing meetings among differences” (Skovsmose 2015, p. 7). In the remainder of this section, we consider some attempts to construct learning situations that permit such meetings.

### 4.1 Concepts of assistance-affording student agency?

As pointed out in Sect. 2, much of the research on low performing students in mathematics has focused on the type of instruction from which students benefit most. Many of these studies have been motivated by ideas about teaching practices that accommodate the students’ special needs or limited cognitive abilities. For example, Milo (2003) pointed out that students with special needs often experience great difficulties in structuring their learning processes, and therefore questioned whether realistic instruction—the prominent Dutch approach on mathematics education—characterized by students’ own contributions in the learning and teaching process, is the most advantageous method for helping these students to learn mathematics. Further studies in special education have also been inspired by the thought that low performing students may be less able to construct their own knowledge in mathematical domains, hypothesizing that these students profit more from a direct instruction approach in which they are taught to use a limited number of specific, proven solution methods (see Peltenburg 2012).

However, the assumption that it is not desirable or perhaps impossible to build further upon the (informal) solution methods that low performing students generate themselves, has some serious consequences (Peltenburg 2012; Scherer 1997). Firstly, restricting students’ developmental space can lead to an attenuation of the richness of the mathematics presented to them. Secondly, guiding students in a rigid way may reinforce the assumption that they are not able to come up with mathematical ideas of their own. McLeskey and Waldron (2011) also expressed concerns about how special programmes can come to restrict rather than afford learning opportunities, pointing to tendencies in such programs including: undifferentiated instruction, a lack of co-ordination with general education, less instructional time, and unclear accountability among the professionals involved. These authors noted the added value deriving from an inclusive education program whose qualities include instruction in small groups; differentiated instructional design, emotional and organizational support, actively supervised independent practice, and progress monitoring.

In conclusion, results such as these suggest that limiting the access of students with mathematical learning difficulties to conceptually rich learning environment might have the effect of continually decreasing their chances to



experience mathematics in ways which make sense to them.

#### 4.2 Substantial and rich learning environments: multiple opportunities

Constructivist and socio-constructivist theories open ways of viewing “knowing” (von Glasersfeld 1995a; Ernest 1994) and learning in a social environment (e.g., Wittmann 2001). For mathematics education, investigative learning and productive practicing are seen as the main elements of these paradigms (e.g., Wittmann 2001). Productive practicing is to be understood in contrast to bare reproduction of knowledge. It should enable pupils to think, to construct and to extend their knowledge (e.g., Wittmann 1990) while being engaged in empirically observable activities and internalising actions and images so that mathematics can, eventually, become a mind activity with numbers far beyond what can be observed empirically as entities or used as ‘manipulatives’ (Flexer 1986; von Glasersfeld 1995b; Streefland and Treffers 1990, p. 315). The teacher has to offer learning situations that enable the students to make discoveries, but this requires that the student possesses powerful tools in the form of (context)-models, schemes, and symbols (Streefland and Treffers 1990, p. 313f).

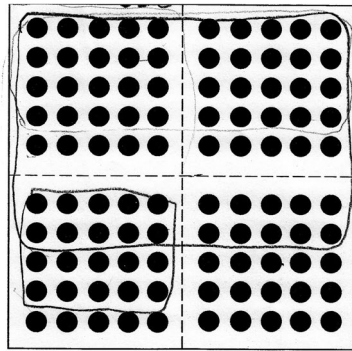
With respect to heterogeneous learning groups, several studies have confirmed that investigative learning combined with productive practicing is appropriate for all learners—*especially* for low achievers and children with special needs (e.g., Ahmed 1987; Moser Opitz 2000; Scherer 1999, 2003; Scherer and Moser Opitz 2010, p. 49 ff.; Trickett and Sulke 1988; van den Heuvel-Panhuizen 1991). According to this view all learners should be confronted with complex learning environments characterised by investigative learning and productive practicing. Such holistic approaches to mathematics teaching and learning require all learners to see relationships between numbers, shapes, and so forth in order to understand mathematical structures (Trickett and Sulke 1988, p. 112). For students with mathematical learning difficulties, however, holistic approaches are often avoided in favour of splitting up subject matter into small fragments.

In this regard, Donaldson (1978) distinguished between the mastery of all the individual patterns or relationships of a system on the one hand, and understanding the nature of a system on the other hand. Splitting a subject into little fragments does not contribute to developing understanding of overarching structural features of mathematics like, for example, understanding the structure of our number system, even though mathematics is often described as the science of patterns and structures. For all students, and especially for children with mathematical learning difficulties, making connections and using relationships could be helpful

for developing understanding (see Scherer 1997). As a consequence, for instance, from the very beginning of the first year of schooling the numbers up to 20 should be offered and dealt with instead of introducing the numbers one after the other (cf. Moser Opitz 2000; Scherer 2013). Taking into account some of the research reviewed in Sects. 2.1 and 2.2, it seems that there still exists scepticism with respect to constructivist or socio-constructivist approaches for students with mathematical learning difficulties. For example, although the results of Kroesbergens’s and van Luit’s (2003) meta-analysis suggest that direct instruction could be the most beneficial type of instruction for these students, this conclusion neglects the fact that students with mathematical learning difficulties profit from teaching specific cognitive learning strategies like self-regulated learning (see Mitchell 2014).

Moreover, to identify children’s existing difficulties, it is necessary to give them the opportunities to show what they are capable of. In this sense, the examples reported in this section can be understood as a plea for on-going change to teaching and classroom practice. Part of this change entails more attention to the creation of substantial and rich mathematical learning environments for inclusive settings, in which different learning trajectories and different forms of interacting with mathematical objects are explicitly recognized (Fernandes and Healy, accepted). The development of such environments is crucially dependent upon differentiation. Learning tasks directed towards levels of difficulty predetermined by the teacher carry the risk that some students are overtaxed or misjudged or fixed at a specific level as viewed by the teacher. Research shows that learning environments that allow *natural differentiation* (ND) can reduce this risk (cf. Wittmann 2001; Scherer and Krauthausen 2010). Natural differentiation means that the learning environment provided is substantial and complex and offers multiple ways of learning and multiple strategies for solving a given problem: the students can choose their level of working by *themselves*, work on several levels of the task and be successful at their level rather than being assessed against one that is predetermined (e.g., Scherer and Krauthausen 2010). At the same time, natural differentiation makes it easier for the teacher to organize the learning processes because all students are working on the same task or problem and there is no need for the teacher to present many different problems.

Consistent with natural differentiation, learning environments allowing own productions or free productions (cf. Streefland 1990) offer various opportunities for students’ use their own strategies and provide their own solutions and thus support suitable differentiation. Examples show that especially students with mathematical learning difficulties often make use of the affordances of such environments and show unexpected competencies (e.g., Scherer



Finde selbst Aufgaben!  
Das Ergebnis soll 100 sein!

$$50 + 50 = 100$$

$$40 + 60 = 100$$

$$50 + -50 = 100$$

$$70 + 30 = 100$$

$$100 + 900 = 2000$$

**Fig. 1** Ali's own productions for findings tasks with the result 100

1999; Grossman 1975). Figures 1 and 2 show the work of two students with special needs (Grade 3) working on the open problem "Find tasks with the result 100". Whereas Ali found only a few tasks just adding the tens, others like Mustafa found 24 tasks representing different types of addends. In general, every learner can work at his or her own level, making use of representations or manipulatives and exploring their competencies.

**Fig. 2** Mustafa's own productions for finding tasks with the result 100

$$97 + 3 = 100$$

$$80 + 20 = 100$$

$$30 + 70 = 100$$

$$79 + 19 = 100$$

$$50 + 50 = 100$$

$$25 + 75 = 100$$

$$75 + 25 = 100$$

$$98 + 2 = 100$$

$$15 + 85 = 100$$

$$20 + 80 = 100$$

$$99 + 1 = 100$$

$$35 + 65 = 100$$

$$54 + 46 = 100$$

$$55 + 45 = 100$$

$$60 + 40 = 100$$

$$45 + 55 = 100$$

$$61 + 39 = 100$$

$$71 + 29 = 100$$

$$10 + 90 = 100$$

$$65 + 35 = 100$$

$$5 + 95 = 100$$

$$63 + 37 = 100$$

$$73 + 27 = 100$$

Similar examples concerning the existing knowledge and active role of the learner are reported by DeBlois (2014).

The benefit of the learning opportunities and corresponding classroom practice described here are related to the potential to meet the individual needs of the students, to provide advice for organizing differentiation in classrooms, to identify specific problems that could be made a subject of discussion for the whole group and to lead to a deeper understanding of mathematical topics. In general, classroom practice should require more than getting the correct result or being able to perform an algorithm but also explaining and reasoning about solution strategies, and considering solution strategies and associated reasoning. Teachers "need to know how to use pictures or diagrams to represent mathematics concepts and procedures for students, provide students with explanations for common rules and mathematical procedures, and analyze students' solutions and explanations" (Hill et al. 2005, p. 372).

## 5 Teacher development in the complex context of assistance of students with mathematics learning difficulties

As the challenge of teaching in inclusive mathematics classrooms becomes more common, the question arises as to what types of professional preparation and ongoing professional development for teachers are most effective, particularly in relation to developing adaptations of the mathematical knowledge being taught to students with mathematical learning difficulties. What initial teacher education and professional development activities are appropriate for supporting teachers in understanding the diversity of learning strategies and learning trajectories amongst students and the kinds of instruments and interventions

that enable all students to engage in conceptual learning of mathematics? According to McLeskey and Waldron (2011), teachers are confronted with the challenges of designing and implementing interventions in the context of a range of complex conditions, including the heterogeneous nature of students' needs as well as significant movement of teachers. In the following section, we consider aspects of teachers' beliefs that are helpful in providing high quality instruction in inclusive settings. As in the preceding sections, the different conditions in different countries have to be taken into account. Having this in mind some of the (partly contradictory) findings seem reasonable and will be commented upon.

### 5.1 The importance of teachers' beliefs

Teachers' beliefs and attitudes towards low achieving children seem to influence their teaching. In a quantitative research study, Abdulhameed (2014) observed particularly that teachers' knowledge of the academic characteristics of pupils with mathematical learning difficulties using strategies correlates positively and significantly with their educational beliefs. In addition, a study by Peltenburg and van den Heuvel-Panhuizen (2012) focused on teachers' expectations with respect to students with special needs. Their study investigated the beliefs of teachers in special education concerning their students' potential in mathematics and what possibilities they saw to reveal this potential. Perhaps a little surprisingly, the data showed that, although the teachers taught students with low achievement scores in mathematics, most of them were positive about the mathematical potential of their students. They often attributed unused potential to causes outside the student and they underpinned this view with observations from school practice.

In contrast to this result, outside of special education, Straehler-Pohl et al. (2014) found that students in a class designated low ability experienced a learning environment characterised by low expectations. In Beswick's (2007/2008) study too, teachers were less inclined to regard the development of conceptual understanding as an appropriate goal for students with mathematical learning difficulties and more inclined to see facility with basic calculations as an appropriate aim for these students compared with students in general. Consistent with this view, these teachers were more likely to consider concrete materials to be tools for getting answers rather than for supporting the development of understanding for students with mathematical learning difficulties, in contrast to their aim for other students. Beswick (in press) found that for teachers the defining characteristic of students considered least capable mathematically was poor basic computational skills and that teachers considered appropriate tasks for these students

to be relevant to their interests and the real world and also easily accessible and not too difficult. Most importantly, the teachers believed that these students needed assistance to develop facility with basic fact recall. In contrast, teachers believed that their most capable students were able to reason mathematically, demonstrate understanding and solve problems and that these students needed to be challenged and provided further opportunities to develop these capacities.

The contrasting beliefs of the teachers reported by Peltenburg and van den Heuvel-Panhuizen (2012) and Beswick (2007/2008; in press) are likely to be related to the differing contexts in which they were working: special education (Peltenburg and van den Heuvel-Panhuizen 2012) and regular, ostensibly inclusive (Beswick 2007/2008, in press) settings. Although further research is needed to establish how these contexts influence teachers' beliefs about the capacities of students with mathematical learning difficulties to learn mathematics, it could be that they provide different frames of reference in which these students' achievements are interpreted. In addition, teachers' choices to work in one or other of these contexts may point to differing underlying beliefs about the value and efficacy of teaching mathematics to students with learning difficulties.

### 5.2 Teachers' mathematical and didactical knowledge

In reflecting on interventions focused on the development of students' conceptual understanding, it is worth considering the mathematical knowledge of teachers as well as other relevant knowledge types. Hill et al. (2005) attempted to identify a relationship between teachers' mathematical content knowledge for teaching and students' achievement gains. They were confronted with numerous challenges, including those pertaining to the influence of students moving between classes during the study and the "imperfect alignment between [the authors'] measures of mathematical knowledge for teaching and measures of students' mathematical knowledge" (p. 383). They were nevertheless able to observe the importance of teachers' mathematical content knowledge in relation to the diversity of students' thinking and to students' 'errors'. However, research shows that different dimensions of teachers' knowledge are related to each other. According to Kleickmann et al. (2015), content knowledge and pedagogical content knowledge constitute distinct but correlated dimensions of subject matter knowledge. For teacher education a broad knowledge (content knowledge, pedagogical knowledge, pedagogical content knowledge) as well as specific didactical knowledge (on "stumbling blocks") is necessary. As discussed in Sect. 4.1, teachers' beliefs about learning mathematics are also of great importance.

DeBlois and Squalli (2001) identified five categories of concerns among professionals working with students with mathematical learning difficulties: errors, students' reasoning procedures, mathematical terminology, the student as person, and the formalization of mathematical knowledge. These authors noted that focusing on errors or students' procedures most often resulted in supplying an explanation or delivering an explicit instruction that lent itself poorly to further adaptation in keeping with didactical (i.e., classroom) factors. Furthermore, focusing on terminology appeared to orient interventions toward identifying words that are meaningful for word problem-solving purposes, but often occurred at the expense of exploring a diversity of possible interpretations and the full range of logico-mathematical relationships at hand. In addition, Pfister et al. (2015b) showed in their video-study range of inclusive classrooms that guiding a classroom discourse and responding in a contextualised fashion to students' inputs is highly demanding. The same could be observed for handling errors productively. Tackling errors and misconceptions by posing leading questions or offering hints to prompt students to identify and correct the errors for themselves was challenging. Addressing these concerns demands that teachers draw upon the full range of knowledge types including mathematical content knowledge, pedagogical knowledge, and pedagogical content knowledge.

### 5.3 Awareness of interactions in classroom

According to the socio-constructivist approaches discussed in Sect. 2, learning mathematics is a social activity. To illustrate the impact of social activities in a classroom, Simmt (2015) proposed a set of markers for observing the movement of a collective—as opposed to individual—learning system, in which classroom learning becomes a place of transformation as opposed to a place of accumulation (i.e., more class members doing the same thing). In particular, she advocated observing patterns of interaction that contribute to the emergence of new, original knowledge. She pointed to the example of a hockey player's "shot" that only becomes a pass if an interaction occurs with another player. The new knowledge can't emerge without an active contribution of each partner—teacher and learner.

To consider classroom interactions, Mary and Theis (2007), whose work focussed on the learning of statistics, have noted how students in special classrooms are faced with several challenges, including inferring cause and effect, putting forward their initial ideas, and negotiating these ideas. Negotiation implies exchange and the possibility of creating new knowledge of relevance to students. An emancipative relationship to knowledge is, therefore, a useful tool for liberating the risk of thinking outside the box of familiar knowledge, in keeping with the new conditions

framing the situation at hand. The nature of the relationship to knowledge, a concept from sociological theories (Charlot 1997), contributes to the development of attitudes where the mobilization of the desire to learn must involve a balance between cost and benefit, and between pleasure and future use. However, we know that a predominance of an utilitarian relationship to knowledge, in which meaning is associated with concrete and practical sense (Beaucher 2010), can reduce the conceptual development of students with learning difficulties (DeBlois 2014). In contrast, Desautels and Larochelle (2003) believed that an emancipative relationship to knowledge could endow learning with a different meaning, in particular by providing learners the opportunity to doubt and pose questions about institutional knowledge. This, in turn, might contribute to students' handling of the surprise of seeing that an item of knowledge does not work in certain circumstances and encourage them to engage with revising their assumptions; empowering them to take the risk of going beyond familiar work methods and previously proven concepts (DeBlois 2015). To this end, socio-historical theories (Vygotsky 1934) could contribute to structure a framework on interactions in classroom.

### 5.4 Additional considerations regarding pre-service teacher education

A number of stumbling blocks can be seen in the professional preparation provided to pre-service special education teachers. Giroux (1999) has noted a tendency amongst these teachers to initially ascribe students' errors to procedures lacking coherence. Following their assessment of the situation, they typically resume instruction via a "normative" adaptation as opposed to a "projective", "withdrawal" or "avoidance" adaptation<sup>4</sup> (DeBlois and Maheux 2005). In the case of pre-service primary teachers who must intervene with mathematical learning difficulties in their ordinary class, DeBlois and Squalli (2002) noted that the same types of intervention tend to occur when they are asked to plan an activity based on students' errors. After analyzing students' statements, these teachers tend to focus on identifying certain specific elements, to the detriment of a comprehensive vision of the situation. Such a response, essentially a one-time intervention chiefly characterized by

<sup>4</sup> A normative adaptation consists in returning to the initial plan whereas a projective adaptation consists in using students' reflections to move forward in the planned content in keeping with students' statements. A withdrawal adaptation consists in allowing students to debate an idea among themselves, in keeping with the view that they will be able to come up with a viable solution. Finally, an avoidance adaptation consists in lowering demands and expectations as a means of encouraging student achievement.

explanation (as opposed to continued questioning or the use of counterexamples), appears to manifest a relationship to knowledge deriving primarily from pre-service teachers' particular past experiences as a student. Johnson and Semmelroth (2014) have confirmed the importance of developing an understanding of students' informal knowledge as well as of the skills required to use this information to assess intervention options—all of which require a theoretical framework for evaluating the creation of questions as well as the validation of the resulting planned activities.

Swain et al. (2012) suggested that pre-service special education teachers entertain certain attitudes having the potential to impact on their attitudes in the classroom. It should be noted that these authors used the word perceptions and not conceptions, considering that their survey was administered after pre-service teachers undertook some experiments with students with learning difficulties. Swain et al. (2012) observed that pre-service teachers recognized that some interventions required nothing more than making minor adjustments in the classroom or that some activities designed for students with learning difficulties were appropriate for regular students, too. Such realizations appear to have transformed these pre-service teachers' perceptions of their ability to work with students with disabilities. In that respect, O'Connor et al. (2015) have explored what they refer to as the "pedagogy of place" and its potential value. Working from the perspective of transformative education, these authors focused on the development of professional identity among pre-service teachers who received training both at university and in a school where they did their teaching practice. They noted a transformation in the attitudes of these pre-service teachers that could help bridge or reduce the gap between practice and theory. For example, the nature of the questions they asked about teaching was different when they were in their practice schools.

In short, developing an understanding of the system in which pre-service teachers operate reveals the need for a framework with which to anticipate, organize and co-ordinate the services to be provided to students with learning difficulties, in particular because they are strongly influenced by their previous experience as previous students. In fact, DeBlois and Squalli (2002) identified three epistemological positions that a pre-service teacher could adopt: previous experience as a student who tried to find answers to their questions, as a university student distanced from his experience as a learner, and as a teacher preoccupied with his students' learning. It is thus vital to propose professional preparation programs that encourage pre-service special education teachers to give consideration to institutional constraints and that prompt them to define their role in relation to the affective, social, physical and conceptual dimensions, so as to foster a variety of adaptations capable of meeting the needs of students with learning difficulties.

## 6 Conclusions and perspectives

In this paper various aspects of the situation of students with mathematical learning difficulties have been discussed. The separation of mathematics education and special education has given rise to specific requirements and problems for research taking into account the different conditions in different countries.

The complexity of the field with respect to definitions and labelling was discussed. Exploring the different ways in which students with mathematics learning difficulties are identified and described in different areas, suggested that many factors can interact to impede the mathematical development of learners, and perhaps, rather than dichotomising learners into those who experience mathematical learning difficulties and those who do not, it might be more useful to adopt approaches to mathematics education that recognise and value the diversity of learners' mathematical experiences, rather than treating differences in learning trajectories as evidence of a deficiency or disorder that necessarily impedes learning or justifies segregation. Taking seriously the ideas of inclusive education and equity that such an approach implies, examples of research focussing on the consequences for teaching and learning processes were illustrated. There is need for more evidence-based research in the field of inclusive mathematics classrooms.

For teacher education programs, first, it is necessary to distinguish between the needs of teachers and needs of pre-service teachers. Faced with the difference of experiences, we must create situations that help pre-service teachers to distance themselves from their own experiences of learning mathematics as school students. In addition, curriculum, beliefs, personal decompression of mathematical knowledge (Proulx and Bednarz 2008) and social activities must be discussed in order to create adaptations for the needs of students with mathematics learning difficulties. The challenge for the teacher is to interpret the events that happen in the classroom in order to make pedagogical and didactical choices. A starting point in constructing a more inclusive mathematics curriculum from this perspective involves envisioning learning scenarios designed to facilitate multiple ways of interacting with mathematical objects and relationships that respect the diverse experiences (sensory, cognitive, socio-emotional and cultural) and identities of the students with whom we work (Healy et al. 2013). Third, teacher education needs to be concerned with developing an appreciation of the discipline of mathematics and of the implications of differing views of the discipline for what it means to teach for all learners and for all learners to be enabled to learn mathematics.

In this paper, the focus has mainly been on students and teachers, but the challenge of providing a quality

mathematics education all goes way beyond the classroom level and involves a rethinking of the institutional structures which mediate both teaching and learning, structures such as curriculum and assessment for example. Experience tells us that it is more efficient to build an accessible building from scratch than to attempt to adapt inaccessible buildings. Can we learn from this as we attempt to build inclusive school mathematics? Perhaps indeed the question is not how we can assist students with mathematical learning difficulties, but how we can learn to build a mathematics education system that no longer disables so many mathematics students.

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