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Attuning to the mathematics of difference: Haptic constructions of number

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CAPTeaM develops and trials activities that Challenge Ableist Perspectives on the Teaching of Mathematics. The project involves teachers and researchers from the UK and Brazil in reflecting upon the practices that enable or disable the participation of disabled learners in mathematics. In this paper, we focus on two themes that emerged from data analyses generated in the first phase of the study: deconstructing the notion of the normal mathematics student/classroom and attuning mathematics teaching strategies to student diversity. Here, we address these themes through exemplifying participants' haptic constructions of number in the context of a multiplication task in terms of four strategies they devise: "counting fingers"; "tracing the sum"; "negotiating signs to indicate place value"; "decomposing".

Keywords: Teacher Education; inclusion; embodiment; ableism.

Inclusive mathematics in an ableist landscape

Educational systems throughout the world continue to be profoundly structured around the construct of the "normal student", a socially constructed student, not a living, flesh and blood person. This construct can be employed to imply that there exists some kind of universal trajectory by which mathematical knowledge can be expected to be learnt, deviation from which is evidence of abnormality and, often, deficiency. Organising the teaching of mathematics according to imposed norms can obscure or even disallow variations in learning associated with different sensory, physical, linguistic, social and cultural experiences and identities – and contributes to a culture in which disability tends to be considered a lamentable condition, a disadvantage that must be overcome (Nardi, Healy, Biza, & Fernandes, 2018). It also results in educational practices developed with students in mind who do not actually exist, rather than for students who will be subjected to these practices.

Our study CAPTeaM (Challenging Ableist Perspectives on the Teaching of Mathematics), aims to challenge beliefs, processes and practices related to mathematics teaching which produce "a particular kind of self and body (the corporeal standard) that is projected as the perfect, species-typical and therefore essential and fully human" (Campbell, 2001, p. 44) and which contribute to the exclusion of disabled learners (e.g., Nardi et al. 2018). CAPTeaM involves inviting practising and future teachers to engage with tasks that encourage them to reflect upon the challenges of attuning mathematics teaching strategies to student diversity and to avoid privileging the notion of a normal student. To this end, we have collected data in Brazil and the UK as participants interact with two different types of tasks.

In the first (Type I), teachers are presented with classroom episodes which show the mathematical activities of disabled students. They are invited to consider how they might enable the engagement of disabled learners within inclusive learning communities. In the second (Type II), small groups of teachers solve a mathematical problem while at least one of them is temporarily and artificially deprived of access to a sensory field or familiar channel of communication.

In this paper, we focus on Type II data and analyses. We begin by outlining the theoretical basis for the task design, which involved linking ideas from the historical-cultural perspective of Vygotsky with aspects of embodied cognition. We then evidence the participants' discursive practices, especially in relation to deconstructing the notion of the normal mathematics student/classroom and attuning mathematics teaching strategies to student diversity. Here, we exemplify said attunement through illustrating participants' haptic constructions of number in the context of a task that invited them to communicate about multiplying a three digit number by a two digit number.

The theoretical underpinnings of CAPTeaM

A major concern expressed by Vygotsky (1997) in his seminal work with disabled learners in the 1920s and 1930s was that the dominant quantitative approaches of his time reduced the question of development to performance on measures that imply deficit not potential. For him, children whose learning is shaped by a disability can be expected to develop differently from their non-disabled peers, but this does not imply lesser development. In a nutshell, Vygotsky's position can be put as follows: if a disabled child achieves the same level of development as a child without a disability, then the child with a disability achieves this in another way, by another course, by other means. For the teacher, he argues, it is particularly important to know the uniqueness of the course along which to lead the child and thus to transform the barriers associated with an impediment into possibilities for development.

Our interpretation of this position (Nardi et al. 2018) is that learning can be defined as participating in, and appropriating (or making one's own), discourses associated with the knowledge discipline we know as mathematics. The process of making something one´s own is shaped by the tools used to act with it – and this includes tools of the body as well as material and semiotic artefacts. Part of understanding the mathematical discourses of learners (with or without disabilities) involves considering how and when the substitution of one (semiotic, material or bodily) tool by another engenders alternative mathematical discourses, which in turn empower the participation of those who have difficulties in interacting with conventional forms. Treating tools of the body as knowledge mediators is consistent with embodied approaches to cognition, which posit that perceptual-motor activities represent a constituent part of our thought processes (Gallese & Lakoff, 2005) and that feeling is part of knowing mathematics (Healy & Fernandes, 2014). Moreover, since that construction and use of all mediational tools have both social and individual dimensions, cognition is as much an interpersonal process as an intrapersonal one.

In teaching, the interpersonal side of cognition is particularly cogent, as it occurs in the context of contact with actions, emotions and senses of others. Indeed, Gallese (2010) has suggested that, when we come into contact with others, our implicit awareness of our bodily similarities result in the activation of the same neural resources when we perceive the actions, emotions and sensations of others as when we experience or execute them ourselves. We accept this suggestion with some caution: not all human bodies are similar and restricting empathy in this way could be used to reinforce exactly the idea of "normal" development that we are trying to avoid. For us, teaching mathematics involves engaging in discourses in ways explicitly aimed at involving learners in sharing the feelings of the teacher about aspects of mathematics, in a process during which the teacher also endeavours to feel the mathematics of the student. This involves a reciprocity of intentions: the teacher attempts to communicate so that her intentions come to inhabit the bodies of her learners, while simultaneously allowing their intention to inhabit hers (Healy & Fernandes, 2014). Given that not all bodies feel things in the same way, this necessarily requires the legitimisation of different ways of expressing and doing mathematics so that difference as well as similarity can be felt as one's own.

This brings us back to Vygotsky and the idea that, as teachers, we need to seek the mediational means that make most sense to the learners we teach and not to expect that the same means will necessarily be appropriable by all – or, even, that the impossibility of using certain tools necessarily impedes mathematics learning. In short, the mediational means that we make available (or not) in learning situations should be attuned to the learners involved.

The aims and methods of CAPTeaM

To explore the role of using different tools of the body in mathematical activities in ways which engage us in recognising and challenging ableism and in developing pedagogies that empower rather than disable learners, we use situation-specific tasks (Biza, Nardi, & Zachariades, 2007). These are research-informed tasks which invite teachers to consider mathematics teaching situations grounded on seminal learning and teaching issues and likely to occur in actual practice (ibid.). Situation-specific tasks can contribute towards generating nuanced accounts of teachers' pedagogical and mathematical discourses as well as facilitate teacher reflection and discursive shifts with respect to how teachers work towards enhancing learners' (disabled or not) opportunities to participate in mathematical activity (Biza, Nardi, & Zachariades, 2018). CAPTeaM involves engaging practising and future teachers with two types of situation-specific tasks, Type I and II, briefly described in the introduction.

Here we focus on Type II data and analyses. Type II tasks are designed with the aim of provoking reflections about how access to mediational means differently shapes mathematical activity. Participants work in groups of three. One member (A) acts as an observer and films the interaction of the other two members. The second member (B) has a learner role and is asked to solve a mathematical problem whilst, temporarily and artificially, deprived of use of a particular sensory field and/or communicational mode (e.g., seeing). The third member (C) has a teacher role, communicating the problem and intervening as judged necessary, but without access to another sensory field or communicational mode (e.g., speaking). In this paper, we focus on one of the Type II tasks (Figure 1).

For the task we consider in the rest of this paper, in each trio (A, B, C), the problem involved multiplying a three-digit number by a two-digit number, e.g., 347x26, although numbers given varied across trios. Then, all convened for plenary discussion of the strategies that had emerged in the small groups. Small-group activity, as well as plenary discussions, were video-recorded. We wish to stress that the aim of the task was not that the participants would attempt to role play the part of someone with a disability. Rather, we would argue that the temporary suspension of a mediation tool that someone is accustomed to use can serve to heighten awareness of alternative possibilities for communicating and expressing mathematics and to encourage participants to consciously attune their interactions according to the particular needs of the other (be they teacher or learner in this task).We chose to constrain the activity of both teacher and learner in the Type II task to highlight the reciprocity of these roles.

Artificially restricting mathematical interactions

For this activity, we will split in groups of three.

One member of the group (A) is the observer.

A second group member (B) will temporarily lose access to the visual field (by shutting their eyes or being blindfolded).

The third member (C) can see but cannot speak.

C will be given a piece of paper with the rest of the instructions.

Instructions to C: Your task is to ask (without speaking) B to multiply 347 by 26 and to indicate whether or not the answer suggested by B is correct.

B should not have access to these instructions.

Once the task is complete, A, B and C have a short discussion about how the restrictions influenced their strategies.

Figure 1. The *Artificially restricting mathematical interactions* Task (Type II).

Data was collected in Brazil and the UK from 91 pre- and in-service teachers (70 from Brazil and 21 from the UK). Bar a small number of in-service mathematics teachers (none with SEND coordinator responsibilities), participants in the UK were students on a Secondary Mathematics PGCE programme. Participants in Brazil included four practicing teachers with some Special Education responsibilities, ten teachers who were also undertaking a two-year Masters in Mathematics Education course, 38 undergraduate students on a four-year course in Mathematics Education (future mathematics teachers) and 18 undergraduate students studying on a four course in Pedagogy (to become generalist primary teachers).

Participants completed four tasks (three of Type I and one of Type II) in three-hour sessions. Data consists of written responses to the tasks (for Type I only) and audio / video recordings of small-group and plenary discussions of the responses. Data collection was carried out once ethical approval by the Research Ethics Committees in both the UK and Brazil institutions had been granted. Analysis of the data aimed to identify participants' perspectives on teaching mathematics to people with different disabilities. The following five themes emerged (see more details in Nardi et al. 2018, p. 154): valuing and attuning; classroom management; experience and confidence; institutional possibilities and constraints; and, resignification.

As we scrutinised the data on each of the above themes, the need started to emerge for robust, factual accounts of the participants' strategies for coping with the tasks. For example, we started asking questions such as what types of bodily involvement do we observe in the participants' interaction? or what communicational channels do the participants deploy during their interaction? In relation to the task in Figure 1, these transformed into questions such as: How do participants communicate about number? How is place value dealt with? Are some numbers more difficult than others? How do participants negotiate ways of communicating Yes/No (Right/Wrong)?

How do participants express, and overcome (if so), any difficulties they experience in this communication? In this paper, we share data excerpts which illustrate answers to these questions and showcase the resourceful ways in which the participants coped with the challenges posed by the task in Figure 1.

Data: Haptic constructions of number and place data

In addressing the aforementioned questions, a suite of strategies emerged that showcase how the participants invented novel ways of doing mathematics, particularly with regard to how they express number when team members B and C cannot see and speak respectively. In doing so, resorting to the communicational channels afforded by the sense of touch – thereafter haptic constructions – became a pivotal characteristic of what the participants chose to do[1]. We exemplify four of these strategies, S1-S4.

S1. Counting fingers. Participants indicate each digit in order, starting with hundreds, then tens and then units, by counting or raising the corresponding number of fingers. Communicating about each digit was easy but sharing the understanding that the three digits were meant as the components of a three-digit number was not. We identified four ways in which the participants coped with this challenge, less or more successfully. Each emerged after the three digits were identified through finger-counting: (1.1) Creating a sign intended to suggest joining the numbers into one. This was generally unsuccessful as it was interpreted by the blindfolded team member as a sign, for example, to add the numbers (Figure 2). (1.2) Continuing directly to indicate the multiplication sign, in a variety of ways (crossing two index fingers or arms, tracing a cross on hand or arm). Usually this had to be repeated a number of times before 3 4 7 became 347 and, even when this was understood, the number tended to be uttered as "three four seven" rather than "three hundred and forty-seven". (1.3) Guided writing of number using the blindfolded team member's hand and a pen or pencil. Finally (1.4), using objects, usually pens, instead of fingers. This was typically quickly abandoned. We return to this in S3 where objects were also used to communicate place value.

Figure 2: Treat the 3 numbers as one (S1.1).

Figure 3. Tracing the written symbol for number on inside of arm (S2).

S2. Tracing the sum. Participants communicate the number as a whole and without explicit attention to place value through tracing on centre of hand, back or arm (Figure 3). We identified three ways in which the participants did so: (2.1) Tracing the written symbol for number on centre of hand, digits signed one after the other on the same location, without indication of the position of each in the whole number and then moving on to tracing the multiplication sign. (2.2) Tracing the complete number on hand, back or arm, with position felt – that is, for 347, the index finger is moved to the

[1] We note that all participants in the role of C chose touch over sound to interact with their partner B.

right as the participant draws hundreds, tens and units – and then moving on to tracing the multiplication sign. (2.3) Guiding hand to write number on paper.

S3. Negotiating signs to indicate place value. Here the digits and the place value are communicated together with two different embodied notations which are used simultaneously in four different ways: (3.1) Placing B's hand in three locations after counting the numbers on the hand (Figure 4). (3.2) Counting fingers on arm, moving the position to different locations on the arm to indicate place value. (3.3) Using objects (screwed up paper balls) and placing on different locations on table. (3.4) Using objects to represent hundreds, tens and units. In 3.4 examples of objects used include pens or the vertical bars on a metal frame door.

Figure 4: Placing hand in different places on the table to indicate place value (S3.1).

Figure 5: Communicating the 300 part of 347 (S4).

S4. Decomposing. This strategy involved breaking down the number according to place value. For example, 347 was communicated as 300 plus 40 plus 7 through finger indication of 3 followed by two 0s, followed by 4 and 0, followed by 7. Figure 5 shows the zeros expressed through forming a circumference with index finger and thumb.

Of the four main strategies, the first two (S1 and S2) occurred more frequently than strategies S3 and S4 in which more attention was given to explicitly representing place value. Generally speaking, these latter strategies emerged in cases in which an S1 strategy was initially employed but those in role B (learner) had difficulty in understanding that a number with more than one digit was involved. Some of the participants in role C (teacher) showed an initial reluctance to change their strategy, choosing to repeat the same pattern of actions in a slightly slower form or by tapping the hand or securing their partners fingers more firmly. We might liken this to repeating commands more slowly, with particular emphasis on certain words. This accentuating sometimes made things clearer, but was more frequently unhelpful. Because the learners were permitted to speak, some chose to explain their needs very clearly. As they asked questions or provided information about their difficulties in interpreting the specific intentions behind the haptic constructions ("*do you want me to add*", "*it could be a 6 or a zero"*), their teachers were motivated to modify – or attune - their strategies accordingly. In some cases, learners explicitly told teachers how to proceed. This was invariably associated with the development of an efficient and effective task resolution. It was also common for the learners to suggest signs for "yes" and "no" (as in "*tap my arm twice for yes and once for no*").

In a small number of cases, the learners seemed reluctant to question or even provide their teachers with details of any interpretation problems. Perhaps they didn´t see this as part of the role of being a learner. Where the teachers were flexible about changing their strategies, this reluctance was not necessarily an impediment to success, and sometimes led to a more vocal participation from the learner as the task progressed.

Least successful were interactions in which the teacher repeated the same strategy and the student only communicated their lack of understanding.

Reflections on alternative mathematical expressions

In our analyses, we consider if and how engaging in this multiplication task motivated the participants to reflect upon how mathematical objects and operations might be expressed in ways that would make sense given the restrictions imposed on both team members C (in the role of a teacher who cannot speak) and B (in the role of a learner who cannot see). We stress that, despite the fact that most of those assigned the teaching role expressed concerns, even desperation, that their task initially seemed an impossible one, in most cases this did not turn out to be the case. Shared signs which enabled successful outcomes generally emerged fairly rapidly. In relation to the strategies S1-S4, devised in the absence of access to spoken or visibly written symbols, objects and gestures were combined in different ways that were gradually attuned to the resources available to the learners. On the way, effective ways of substituting temporarily disabled channels were invented.

This process of attunement drew heavily on what the teachers knew about the learners' previous mathematical experiences, and all of the strategies that were employed appeared to be directed at enabling the learners to re-enact previously experienced mathematical practices – albeit by activating expressive forms not commonly associated with multiplying numbers. Place value was not being introduced to the learners, it was being triggered through haptic means. The different haptic realisations of number allowed, eventually, the learner to feel the intentions of the teacher. This however was not always immediate, as it required some time for the teacher to accustom to simultaneously inhabiting a body which could not speak (their own) and a body that could not see (the learner's).

As described above, the attempts of both teachers and learners to appropriate each other's intentions were facilitated when the participants in the role of learner also assumed some of the responsibility for communication. It was also common for those in the role of learner to assign to the sighted teacher the task of remembering numbers that the blind learner could write down but could not see, here the other becomes a substitute tool. In the group discussions, these were issues that the participants highlighted as they reflected on how the temporarily imposed restrictions opened windows on pedagogical strategies that might be employed with disabled students. The idea of giving the student a role in guiding the teacher was one approach suggested:

> I was thinking before [about teaching a blind student], I would be lost, I wouldn´t know what to do to teach someone who is blind. But you have to listen to the person. It was her who showed me the way, in this case blind, she gave me the way. "Do it like this, do it like this". She gave me a way of communicating with her.

By requiring diverse forms of bodily involvement, Type II tasks provide opportunities for participants to consider the many and varied ways in which a mathematical problem can be approached. They permit a moving in and out of their long-established mathematical and pedagogical comfort zones, and a growing appreciation of difference as well as the enactment of agency shifts that these moves may imply. These are attributes of mathematics teaching that are pertinent at large and by no means exclusive to the teaching of disabled learners.

We see CAPTeaM tasks as inviting us to attune our teaching strategies in ways that harness the different potentials of different students, and to not associate mathematical ability with fluency with mediational means that are not available to all.