EMBODIED MATHEMATICS: RELATIONSHIPS BETWEEN DOING AND IMAGINING IN THE ACTIVITIES OF A BLIND LEARNER

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In this paper, we examine the claim that mathematical cognition is embodied by exploring the co-ordinations of speech, gestures, material objects and sensory activities in a dialogue between a mathematics teacher (researcher) and a blind student. We argue that the student came to know aspects of the mathematics in question (symmetry and reflection), in a process involving the mental simulation of past experiences in ways which enabled associations between physical and mathematical activities. In particular, we suggest that the researcher had a role in facilitating a kind of meeting between culture and cognition by inviting the learner to make connections between sensory experiences (past and present), representational artefacts and culturally accepted mathematical meanings.

EMBODIMENT, MEDIATION AND MATHEMATICS LEARNING

Evidence to support the idea that cognition is embodied is growing (Barsalou, 2008; Gallese and Lakoff, 2005), adding force to the argument that the way we think cannot be separated from the way we act and that both have their bases in our body, its physical capacities and its location in space and time. Embodied approaches emphasising how even the most abstract of symbols have physical grounding have also begun to permeate research in mathematics education, (Nemirovsky and Borba, 2004; Lakoff and Nunez, 2000; Radford *et al.*, 2005). Yet, given the importance of symbol systems in accessing, or perhaps better in moulding, mathematical activity, research in mathematics education which draws on embodied cognition tends to do so within a framework which also takes into account the centrality of tool mediation, from an essentially Vygotskian tradition, in the process of meaning making (see for example, Arzarello et *al.*, 2009; Radford *et al*., 2005).

The attempt to combine embodied cognition with semiotic (and indeed material) mediation characterises the approach we are adopting in our own work with students who lack access to one or other perceptual field. We too have become increasingly convinced by the argument that our mathematical understandings are structured by our encounters and interactions with the worlds we experience via our bodies and our brains (Gallese and Lakoff, 2005). At the same time, we are also aware that mathematics as a discipline is a cultural affair, with learning involving a delicate process in which learners become increasingly aware of how their subjective

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meanings for a given mathematical object connect to the cultural meanings emphasised in teaching. Radford (2002) calls this the process of objectification. In this perspective, thinking involves the coordination of speech, body, gestures, symbols and tools. Moreover, Gallese and Lakoff (2005) argue that circuitry across brain regions link different sensorial modalitities "*infusing each with the properties of others*" and claim that imagining and doing use a shared neural substrate, offering neurological evidence for the view that thinking is inherently multimodal. In Healy and Fernandes (2009), we presented evidence of the multimodal nature by which a blind student and his teacher gradually communicated their developing conceptions of a three dimensional shape, suggesting that the gradual and dynamic exploration of the solid by the student, who employed his hands as tools for perceiving, led him to notice rather different aspects of the object than are usually highlighted by the synthetic visual feedback received when the eyes are the seeing tools in question. This result indicates a need for more research to investigate the specific ways in which the sensory canals through which we make contact with mathematical activities shape the mathematics meanings that become associated with the objects in question.

Such research is also critical if we are to become better able to intervene in the learning processes of these students and to design effective instructional situations. It is these issues that motivate our current research programme which aims to (1) *investigate forms of accessing and expressing mathematics which respect the diverse needs of all our students*; (2) *contribute to the development of teaching strategies which recognise this diversity;* and (3) *explore the relationships between sensory experience and mathematical knowledge.*

In this paper we concentrate mainly on the second two questions as we analyse the interactions between a blind student, who we call Edson, and a researcher (the first author) who assumed the role of his mathematics teacher during an instructional situation related to axial symmetry.

THE RESEARCH ACTIVITIES

The learner who participated in the experimental activities discussed in this article was not born blind. He lost the sight in his left eye at the age of four years. Following thirteen surgical attempts to save the sight in his right eye, at the age of fifteen, after suffering a dislocation of the retina, he lost all but two percent of the vision in this eye too, leaving him with the capacity to distinguish only between light and dark and to identify the direction from which light is coming. During the time that Edson participated in this research project, he was a student in the third year of High School, studying at night in a public school in the state of São Paulo. For his mathematics lessons, he was included in the regular classroom, and although Braille translations of the exercises were provided for these lessons, he received no extra additional support. During the day, Edson worked as a receptionist.

Edson participated in three sessions involving activities related to axial symmetry. The first session of 1 hour was dedicated to activities involving identifying if figures are symmetrical and locating their axes of symmetry. In the second session (1 hour and 30 minutes), he worked on constructing reflections of figures in given lines. The third session (1 hour 30 minutes), revolved around reflection as a mapping of the plane onto itself and the proprieties preserved in this mapping. All the sessions were video recorded and the recordings transcribed for analysis.

Before working on the mathematical activities related to symmetry and reflection, Edson participated in an interview during which he talked about his visual memories. He explained that not only did he have visual memories, but that he used them to recognise shapes, now experienced through touch, that he had learnt about when he could still see. Edson's description of the process by which he recognised these shapes suggested to us a certain resonance with the claim that thinking is multimodal introduced above, motivating us to invest further in explorations of the embodied cognition literature. Barsalou (2008) argues that it is our perceptive experiences that enable us to form the representations that are stored in our memory. These representations are subsequently reactivated to simulate the perceptive, motor and introspective states associated with an object, even when it is not present. That is, the states associated with past experiences of a category of knowledge are relived through simulation. Our interpretation is that it is through this process that an individual is able make present, through a temporal activation, cerebral processes created during past experiences. Given the strong influence that Vygotsky has had on our developing work, as we began to consider how the ideas about the role of simulation in cognition might relate to mathematics learning and hence how they might be exploited in instructional situations, we were reminded of his view of the developing child. Characteristically, in Vygotsky's version, which also emphasises temporal activation, language is attributed prize of place:

> …the child, with the help of speech, creates a time field that is just as perceptible and real to him as the visual one. The speaking child has the ability to direct his attention in a dynamic way. He can view changes in his immediate situation from the point of view of activities, and he can act in the present from the viewpoint of the future." (Vygotsky, 1978/1930, p. 35-36).

What Vygotsky appears to be arguing here is that the child becomes able to act in the present, drawing on the experience of past activities while attending to an endpoint, or goal, for the future. In the context of instructional activities, it is the role of the teacher to support exactly this kind of activity, so that a zone of proximal development (ZPD) might emerge in which the learner can use what he/she already knows (results of the past) to realize in the present (results of today), with teacher guidance, an activity that will come to characterise his/her own future performances (results of tomorrow).

With these ideas in mind, we now return to the instructional situations in which Edson worked on a series of tasks related to reflection. He had not studied geometrical transformations before and the term "axis of symmetry" was unfamiliar to him at the start of the research – although he had come across, at least nominally, some other elements that were fundamental in the developing dialogues, including midpoint, perpendicular lines and angle bisector. Our aim is to consider how the various resources of the setting contributed to Edson's growing awarenesses of the mathematics of symmetry and reflection.

Towards a meaning for the axis of symmetry

During the first session, the initial activity involved the determination of the axis or axes of symmetry of two dimensional figures cut out in card. At this point, a figure was deemed to be symmetrical if it could be folded in half in a way that one half coincided completely with the other – a definition offered by the researcher. The fold that divided symmetrical figures into two congruent parts was indicated as the axis of symmetry.

The next set of tasks consisted of determining the axis of symmetry of figures represented by elastic bands on a wooden geoboard, divided into a grid by nails (Figure 1). Given the change in the way the figures were represented, it was no longer physically possible to locate a potential axis of symmetry by folding. However, as the following discussion illustrates, Edson was able to correctly locate the two axes of symmetry for the hexagon he was working with, seemingly by mentally simulating the process of folding:

Edson: *Here is one axis of symmetry. Right?* (he traces his figure along the vertically placed dividing elastic). *Here we have more* (traces the elastic dividing the hexagon horizontally). *Two right? It has two axes of symmetry.*

Figure 1: Edson simulates the act of folding

Res: *Two axes of symmetry. It doesn't have any more?*

Edson: *I am doing it in my head. It's not possible to do any more*

Res: *No, why not?*

Edson: *First, I used touch to get an idea of the drawing. Now I am using my head, as if I was folding it with my hands.*

Res: *You're imagining the figure*?

Edson: *Yes, visually. Here one* (at this point Edson makes a gesture to indicate folding around the horizontally place axis as shown in Figure 1). *And here two* (uses a similar gesture to simulate a fold on the vertical axis). *It just has the two.*

Edson's description indicates that by touch he created an idea of the hexagon, which for him was experienced as visual. He then imagined that he was folding the shape represented on the wooden board. While we cannot see what parts of his brain were activated in this process, it does not seem unreasonable to hypothesise that the same neural substrate might have been involved in both the actual folding action and when folding is imagined. Certainly, it would seem that the location of the axis of symmetry was a multimodal experience for Edson, although we don't really know what he sees/feels when he uses the term 'visual'. Additional evidence for the claim that doing and imagining evoke the same kinds of cerebral activity is provided in the form of Edson's gestures, in which the act of folding the imagined figure is actually mimicked. Folding is hence more than a specific action on a concrete object; it is a general procedure (which Gallese and Lakoff would call a schema). Moreover, we would argue that, at this point, Edson's idea of the axis of symmetry is intimately linked to the multimodal representation involved in the folding process.

It also would seem that the teaching strategy of suggesting the association a fold in a symmetrical figure with the axis of symmetry was an effective one for promoting learning in this case. To use Barsalou's (2008) terms, we might say that it permitted the activation of resources stored in the memory formulated in the recent past, when he was working on the paper figures, which not only enabled him to solve the (present) task, but also emphasised the congruency of the shapes either side of the axis, important for the activities which were to come (future). In this case, we argue, it was the physical activity of folding that was instrumental in the emergence of a ZPD in which Edson extended his knowledge of geometrical figures to include the beginnings of a notion of axial symmetry.

Attending to the proprieties of reflection

The next set of tasks was introduced in the second research session. The aim was to move on from the considerations of symmetrical figures to the study as reflection as a transformation. By this point, as we have said above, Edson already seemed to have appropriated the idea that the figures either side of the axis of symmetry were to be congruent. In all the attempts to create images of a given figure under a reflection, the images (here we use the word 'image' in its mathematical sense) that he represented on the geoboard were congruent to the originals. He did not however always preserve the other image properties; neither necessarily attending to the reversal in orientation of the points in the image as compared to the original figure nor the placing all the image points so that their distance from a point on the axis was equal to the distance of the corresponding points of the original figure to the same point.

A discussion of the issue of inversion emerged as Edson attempted to represent on the geoboard the image of a scalene triangle in a vertically placed axis (Figure 2). Although he successfully completed the task on his second attempt, as he checked his solution, he focused only on verifying the congruence of the two triangles. In an attempt to get him to articulate the inverted nature of the image, the researcher intervened, asking him about possible previous experiences with mirrors:

Res: *In your visual memory, do you have your image in a mirror?*

Edson: *Yes.*

Figure 2: Reflecting a triangle

Res: *And how did you see your image in a mirror?*

Edson: *Like how?*

Res: *... Pretend that I am your image in a mirror. I am in front of you. Raise your left hand. If I was to touch your hand, which hand would I have to raise?*

Edson: *You're right. I get it, it's inverted.*

In this exchange, the researcher attempted to call Edson's attention to a particular property associated with the reflection transformation by inviting him relive an experience associated with his visual past. In this case, the mental simulation was not a spontaneous act on him part, but a suggestion from the researcher. Nonetheless, the evoking of his previous experiences with mirrors still seemed to favour the use of previously stored memories in an imaginary simulation of an activity that was no longer physically present for Edson. The intervention of the researcher in this case contributed to the reactivation of the multimodal representation structured during a past experience – that of seeing himself in a mirror – simulating the states associated with this act. This helped Edson to (re)experience the inverted nature of images in a mirror. We might say, in Vygotskian terms, that the invitation to participate in the act of imagining a past experience contributed to the emergence of a ZPD in which Edson's view of reflection moved closer to the view the researcher wished to emphasise.

It was in a similar manner that the researcher tries to bring to Edson's awareness the invariance under reflection of the points along the axes of symmetry. This occurred as they discussed Edson's incorrect attempt to produce the reflection of a segment in a diagonal line (first photo in Figure 3).

Figure 3: The image of a line segment

Res: *When you have an image in a mirror, if you put your finger touching the mirror, would you have its image touching the mirror too. Finger to finger?*

Edson: *Yes*

Res: *Here, the axis of symmetry is as if it was the mirror and this point here like your finger.*

Edson: *Ah, this is wrong (*Edson reconstructs the image successfully as shown in the second photo of Figure 3).

SOME CONCLUDING REMARKS

In presenting extracts from the dialogues between Edson and the researcher, we have tried to show the importance of his embodied experiences, both past and present, in the process of objectifying knowledge related to symmetry and reflection. As the research sessions unfolded, both participants became involved in a mutual coordination of the resources available in the setting in order that Edson might appropriate those cultural meanings associated with the mathematics involved deemed important by the researcher. The resources available were multiple and manifested in a variety of different forms. They included the tactile representations of geometrical figures especially structured to enable the exploration of geometrical objects by blind students. These took the form of cut-out foldable figures and the wooden geoboard with line segments and points that could be discerned by touch. In the dialogues between researcher and learner, a number of additional resources were also evoked. The use of language and gestures was critical for the communication between the two and represented the basis for our analyses. We argue that they structured the discussions in ways that brought into play not only the resources present physically but also the multimodal representations stored in the memories of both participants from their previous interactions with their respective worlds.

In the first session, symmetry began to become associated with the deconstructing of a single figure into two congruent parts, separated by an axis of symmetry. And it was the activity of folding, realised either physically or in an imaginary simulation that underpinned the meanings that were expressed in the activities of this session. It is important to stress that it was a rather specific kind of folding that was involved, as it is not always that we fold to create congruent parts. This seemed to be quite clear to Edson and we note that, in a discussion which occurred after the second session of activities, he made a connection between folding clothes, and in particular his bedcover, that could be associated with symmetry, suggesting that the fold is made "*point to point, it has an axis of symmetry … actually possibly various axes of symmetry*". Of course, there are a number of limitations in associating folding with reflection – it emphasises symmetry rather than reflection, in that the relationship of dependence between the original points and their images is not highlighted. But this does not mean that in the multimodal representations associated with the mathematical object that we store in our memories, we wipe out any connections with the physical in favour of some kind of disembodied symbolic representation.

In the second session, it was by connecting the interactions with Edson's past experiences of mirrors that he became aware of some of the mathematical proprieties inherent in the activities he was working on. Once again, we suggest that this led to embodied forms of conceptual knowledge. It was he himself who suggested that, although he could no longer see, he still made sense by imagining visual experiences from the past. It was these memories, coordinated with the tactile explorations and the ongoing discussion, that we believe were behind the meanings for the transformation reflection that emerged during this research session. And, given our research has its roots in socio-cultural theory, we cannot forget the extent to which these meanings were culturally constituted. In both sessions, the researcher had a fundamental role in guiding the explorations towards culturally accepted meanings and, also in both cases, it was she who essentially motivated the associations between past and present experiences in ways intended to encourage Edson in the building of appropriate mathematical models for future activities. That is, in contributing to the emergence of a ZPD where culture met with cognition. In this case, the researcher had the option of appealing to Edson's visual past. Our next step will be to analyse interactions in similar instructions scenarios with learners who have been blind from birth and to explore what kinds of interventions and associations are appropriate so that these learners engage in the process of objectifying geometrical notions.

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