THE ROLE OF GESTURES IN THE MATHEMATICAL PRACTICES OF BLIND LEARNERS

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Our research aims to explore the mathematical practices of blind learners. To contribute to the overarching question of how our physical senses mediate our interpretation of mathematical phenomena, we seek to identify how learners without access to the visual field make use of the various resources in learning settings, both physical and semiotic, to negotiate mathematical meanings. In this paper, we concentrate our analyses on the mathematical practices of one student, blind since birth, as he attempts to make sense of activities involving perimeter, area and volume. We consider in particular the role of his gestures, not only in the dialogues established with others, but also in structuring his own mathematical ideas.

INTRODUCTION AND BACKGROUND

On the 7th of July of 1688, William Molyneux sent a letter to John Locke in which he proposed a question that caught the interest of various philosophers of the time and continues to generate discussion until today. The question raised by Molyneux was whether a person, born blind, who had learned to distinguish between a sphere and a cube through touch, would be able to correctly identify these solids by visual means if he or she gained to capacity to see (Riskin, 2002). Not surprisingly, positions were divided as to the correct response to this question, with the different responses reflecting the different values attributed to experiences originating in the senses and perception, and the influence of the body on cognition. Given the particularity of the subjects involved in our research, blind school students, we have a special interest in this relationship.

A more radical version of Molyneux's question was considered by Condillac, who elaborated a sensualist theory of knowledge in the work *Treatise on Sensations* published in 1754. (Condillac and Degérando, 1989). By simulating the process of humanisation of a marble statue, he considers what this being would come to know if each of the senses were to be acquired in isolation from the others, or in combination with one or two others. In this way, he presents his thesis that all knowledge evolves from the transformation of sensations, or from what we could call perceptions.

Amongst the various philosophical perspectives which discuss the relationship between perception and knowledge, phenomenological theorists argue that perception is primary in human knowledge. For phenomenologists, there is no difference between sensation and perception, because there is no such thing as a partial or

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elementary sensation (Chauí, 2000) – we see and we perceive forms, that is, structured totalities seeped in sense and in meaning. In this way, perception, or perhaps better, the perceptual field, is key in the complex relations between our bodies and our world , or rather, between the subject and the objects in a field of visual, tactile, olfactory, gustatory, auditory, spatial, motor and linguistic meanings (ibid.). If phenomenologists are agreed on the cognitive function of bodily activity, one divergence which distinguishes between the positions of two influential thinkers, Husserl and Merleau Ponty, relates to the importance of the sociocultural. The position adopted by Merleau Ponty is that "senses grouped by the same name are experienced in distinct and even contrasting manners by people from different cultures" (Furlan e Bocchi, 2003). What attracts us in this perspective is the recognition of the influence of sociocultural factors in the constitution of semiotic signs and systems.

Turning more specifically to mathematics education, Nemirovsky and Ferrara (2005) also emphasise the cognitive importance of the body in mathematical thinking. For them, understanding of a mathematical object is intrinsically linked to the ways in which the tasks given to learners motivate different areas of perception, stimulating changes in states of attention, conscience and emotion which bring a perceptual-motor character to mathematical understanding and thinking (Nemirovsky, 2003). It should be clear by now that, like them (and many others), we do not think of cognition as something confined to cerebral activity and that we believe it is not possible to disassociate experience and perception from cognitive activity.

As we posit a centrality to bodily experiences in cognition and accept that physical senses play an important role in interpreting mathematical phenomena, an interesting question is how those without access to particular senses interpret the behaviour of mathematical objects. If we can identify the differences and similarities in the mathematical practices of those whose knowledge of the world is mediated through different channels, perhaps we can gain more robust understanding of the relationships between experience and cognition more generally. The blindness, or the absence of a particular sense, of the learners with whom we are working leads us to concentrate our attention on *tactile practices* – touch being a sense rather neglected in studies of mathematics learning – and *semiotic practices* – such as spoken language, Braille, diagrams, graphs, gestures for example. Together, it is through touch and through such semiotic practices that blind learners express and reveal their ideas, intentions and emotions.

We know that there are some differences in the ways that blind learners process data when compared with sighted students. When exploring an object, for example, the hands of the blind learner, like the eyes of the sighted, although in a slower and successive form, are moved in an intentional manner, catching particularities of the form in order to perceive – and at the same time conceive – the object. Touch permits a gradual analysis, from parts to the whole, whereas vision is synthetic and global. The partial information supplied by touch has a sequential character and must be integrated, constructed into a coherent whole. One question, then, is the extent to which this form of exploration highlights particular mathematical relationships and properties or particular ways of thinking about mathematical objects. A second question relates to language, or more precisely, the role of gesture in language. Recent research in mathematics education and beyond (see, for example, Nemirovsky, Roth, McNeill, Iverson and Goldin-Meadow) has considered the potential – both communicative and cognitive – of the spontaneous movements of the body, hand gestures, noddings of the head, changing of postures and the like, which accompany discourse. But what of blind learners who cannot see the gestures of others (or, being the case, their own)? Are gestures really important for all in mathematics learning?

In the rest of this article, we concentrate on this question, exploring the role of gestures in the discursive practices of a blind learner when the objects of study are volume, area and perimeter. Returning to the phenomenological perspective of Merleau Ponty, in which knowledge comes from experiences with the body through its existence in a world which is temporal and spatial, it is important to take account of the context in which the interactions we analyse occurred. The discursive practices used by the learners and researchers, including their use of gestures, were enacted with the intention and belief that they are shared means of discussing school experiences contextualized by the cultural instruments related to the same. The particularities of the gestures presented in the text that follows are shaped, not only by the specific characteristics of the learners, but also by the mediational systems (the material tools and discourses), detailed in the next section, that permeate the instructional scenario.

THE RESEARCH CONTEXT

During twenty-seven months, we worked, in collaboration with a group of five school teachers, on a research project aiming to investigate the processes by which blind learning appropriate mathematical knowledge and to design learning situations conducive to these processes[1]. The project took place in a school from the public school system of the state of São Paulo in Brazil, with a long history of including learners with visual impairments. During the project, a total of twelve blind or partially sighted students participated in the various empirical activities. The study of volume, area and perimeter considered in this paper occurred over four research sessions, each of approximately 90 minutes. Four students, all congenitally blind, and two researchers took part in these sessions. In the first two sessions, the activities centred on the area and perimeter of plane figures (starting with quadrilaterals), the third initiated work on volume and in the final session, the students worked on a task which involved determining the most economical amongst a range of boxes and other rectangular prisms. All the sessions were video recorded. Three different material

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tools were elaborated for the activities, all of which were intended to favour tactile exploration. The way in which the tasks were proposed and the organisation of the students aimed to stimulate dialogues between the participants, between the researchers and individual learners, between pairs of learners and between all participants (four learners and two researchers). The choices related to task design were consistent with Nemirovsky's views on perceptual-motor activity in that we sought to activate two perceptive channels of the learners – audition and touch and to incite the communication of their perceptions by means of speech and gestures.

For this article, we have selected, transcribed and codified episodes from the first research session to show how one of the students, Leandro (14 years olds) made use of the semiotic resources of the learning scenarios to create mathematical meanings which then mediated his practices in the latter sessions. In our analyses, we seek to illuminate and understand the role of gestures as both instruments for communication and cognition. With this is mind, we applied the classification of gestures proposed by McNeill (1992):

Iconic gestures (♦) have a direct relation with the semantic discourse, that is, there exists an isomorphism between the gesture and the entity it expresses. However, comprehension of the iconic gesture is subordinated to the discourse which accompanies it.

Metaphoric gestures (●) indicate a pictorial representation of an abstract idea which cannot be represented physically, for example, when we illustrate with hand movements the limit of a function f(x) when x tends to zero.

Deitic gestures (♣) have the function of indicating objects (real or virtual) people and positions in space.

Beat Gestures (♪) are short and rapid and accompany the rhythm of the discourse giving special meaning to a word, not due to the object that it represents but to its role in the discourse.

Our work and our analyses are very much at the exploratory stage – as there are currently very few studies which discuss the role of gestures in instructional scenarios composed of blind participants from which we can draw. Goldin-Meadow (2003) and Iverson and Goldin-Meadow (1998) suggest that blind individuals make as much use of gestures as the sighted, at least as far as beat, deitic and iconic gestures are concerned, using this as evidence to support their cognitive as well as communicative function. Our objective however is not only to classify gestures, but to understand how, in concert with other resources within instructional settings, they become tools for creating and communicating mathematical meanings.

Creating a gesture to represent area

Since the mathematical concepts under study during the research sessions are concepts usually developed in elementary as well as high school mathematics curricula, at the beginning of the first session, we asked Leandro to describe his knowledge about area and perimeter. He offered the following definitions:

"*Perimeter is all sides. It's the border of the figure. Area is the internal space"* At this point, his discourse was not accompanied by any gestures. Leandro's description suggests a certain familiarity with the mathematical terms, however, it seemed at the beginning of the task that his expressions were echoes of the voices of others from previous activities and when Leandro was given a board with impressions of four rectangles (two of which were filled with unit cubes as shown in Figure 1), he was not able to determine either perimeter or area. Indeed, Leandro, along with the other students, explained that it was very rare for them to interact with representations of geometrical shapes, usually they were given the measures along with an appropriate procedure – their task was to calculate a numerical result.

Following an intervention from the researcher who indicated the perimeter of a rectangle (measuring 8cm by 3cm), by tracing Leandro's hand over the edges of the form and the area by passing his hand over the cubes that filled the shape (Leandro is the student in the orange jumper in the left of the figures which follow), he quickly determined that the sum of the four sides was 22, but was still not sure about the area:

Figure 1: Adding the sides.

Figure 2: "Eight, eight, eight"

Leandro: *The perimeter of my figure is 22. It's this, all this from here, isn't it?.* (♣) (He retraces his fingers around the sides he has added, a deitic gesture intended to enable the sighted researcher to see that he had appropriated her strategy)

Res: *And the area?*

Leandro: *...I don't know.*

Once again the researcher took Leandro's hand and moved it over the internal space of the rectangle, this time repeating the words originally offered by him. After some exploration, Leandro uses a beat gesture to indicate he has arrived at the solution:

***Leandro**:* *So, my figure has 24.* (♪) (Beats one of his hands twice on the figure)

***Res**:* *And why do you think it is 24?*

***Leandro**:* *Because it has 24 little squares here* (♣) (Places his hand on the figure).

***Res**:* *And how did you work it out?*

***Leandro**:* *I did 8, 8, 8, gave 24* (♦) (Traces his hand from left to right over each row of eight cubes which compose the area of the rectangle).

We have classified Leandro's last gesture as iconic and not deitic since the actions of his hands accompanying his speech not only indicate the objects in question but also simultaneously compose the area being measured. This gesture turned out to be central to the rest of his activities – not only for area but also for volume. As we were hoping that the students would elaborate methods, meaningful for them but also with a general applicability, the next task given to Leandro was to calculate the measures for a rectangle of 8cm by 5cm. This time, he did not have access to any external concrete representation of the figure although did still have access to the wooden centimetre cubes.

Leandro rapidly calculated the perimeter of the rectangle, again making use of an iconic gesture in which he traced the perimeter over the imaginary sides of the rectangle on the table in front of him (Figure 3). Clearly, although he could not see it, this gesture was directed at himself, perhaps the sighted learner would have drawn a rectangle at this point, but sketching a figure is a difficult option for the blind learner and, for this task at any rate, the physical activity of tracing out the generic rectangle was sufficient for Leandro to perceive its perimeter.

Figure 3: Calculating perimeter (♦)

Figure 4: Hands as rectangle (♦)

Once again, Leandro found the calculation of the rectangle's area more demanding. Given his apparent difficulty, the researcher suggested he explained his thinking:

Leandro: *I am thinking, if I do 8 here and 5 here* (♦) (positioning his hands as shown in Figure 4). *Then I could...*

Figure 5: An external sign

With this gesture, Leandro uses his hands to represent two of the sides of the rectangle, but at this point, the sign created is not enough to enable him to see the calculation that he could employ. Instead he returns to a strategy involving the material tools and positions on the table an L-shape formed of eight cubes arranged in a horizontal row and five in a vertical column (Figure 5). In this way Leandro creates for himself a sign which enables him not only to calculate the area of the given shapes, but also to appropriate a general method, which involves decomposing the figure into rows.

Leandro: *I made a little line with eight and one with five* (♣) (places his hand on the L-shape), *so the perimeter is 26. For the area, we need to complete. I made as if I was completing it. I did eight times five* (♦) (traces on the table five imaginary lines of eight cubes) *which gives 40*.

From this point onwards, over the rest of the four research sessions, Leandro consistently used the gesture of tracing imaginary lines (in the air or on a surface) when he talked, or as he thought, about area. Clearly this gesture for area was strongly shaped by the material tools made available during the first research session, as well as by the initial interventions of the researchers. In the third session, when he began to work with volume, he generalised this gesture: instead of tracing imaginary lines with one finger he moved his whole hand as if creating a series of layers to compose the volume of a three dimensional shape.

REFLECTIONS ON OUR ANAYLYSES

What we have tried to illuminate in our analyses is how the blind learner is able to create a sign which represents his physical experience, his perception of a mathematical object, by coordinating the various resources available in the learning setting. In this short example, we have focussed mainly on the interactions of one blind learner with a sighted researcher. Leandro knew the researcher could see his gestures, but he also made gestures to himself, a strong indication of the cognitive importance of this physical activity. Indeed, our data indicates that when communicating with other blind learners, Leandro continued to use gestures, pointing at objects and making hand movements, that neither he nor his partner could see to structure his own thinking. On some occasions, he would subsequently take the hand of his partner and re-enact his gestures so that they could be both communicated and physically experienced by another blind learner.

Our analyses thus far convince us that gestures are as much a part of blind learners' mathematical practice as of the sighted. Indeed, iconic gestures, such as those developed by Leandro, may actually be more important to the blind, since we suspect that a sighted learner may have chosen to sketch the generic rectangle, rather than gesture it. In the example presented in this paper, metaphorical gestures did not appear to play a part in the problem solution. Since Goldin-Meadow (2003) has also alluded to a relative absence of metaphoric gestures amongst blind individuals in her studies, this is a finding that merits further exploration. However, the favouring of iconic gestures in Leandro's practices in the these activities may be related to the concept under study – area is something relatively straightforward to represent physically – and may also be related to the specifics of structuring of the task and materials with which he was working. And this brings us to another important question for exploration. To date, the learning settings we have investigated have involved adaptations of situations originally planned for sighted learners. Inevitably, these learning situations were designed on the basis of what we know of the learning trajectories of sighted learners. It may be that the gradual processing of data that results from tactile as opposed to visual exploration makes possible rather different trajectories to mathematical knowledge that we are not yet exploiting – and that may help in the design of new learning situations that contribute to changing the mathematical experiences of a variety of learners and which respect the diversity and potential of different forms of accessing mathematics.

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